4 Determinants

Fastrack Revision

- To every square matrix of order n, a number can be associated which is called determinant of the square matrix.
- For matrix A, $|A|$ is read as determinant of A.
- The determinants of non-square matrices are not defined *i.e.*, in each determinant:

  Number of rows = Number of columns
- $\begin{vmatrix} a_1 & b_1 \\ a_2 & b_2 \end{vmatrix} = a_1b_2 - a_2b_1$
- $\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{vmatrix} = a_1\begin{vmatrix} b_2 & c_2 \\ b_3 & c_3 \end{vmatrix} - b_1\begin{vmatrix} a_2 & c_2 \\ a_3 & c_3 \end{vmatrix} + c_1\begin{vmatrix} a_2 & b_2 \\ a_3 & b_3 \end{vmatrix}$

  $= a_1(b_2c_3 - b_3c_2) - b_1(a_2c_3 - a_3c_2) + c_1(a_2b_3 - a_3b_2)$

  $= a_1b_2c_3 + b_1c_2a_3 + c_1a_2b_3 - a_3b_2c_1 - b_3c_2a_1 - c_3a_2b_1$
- In a determinant of order n, number of elements $= n^2$
- If A and B are square matrices of same order, then

  $|AB| = |A||B|$
- If the coordinates of the vertices of a triangle are $(x_1, y_1), (x_2, y_2), (x_3, y_3)$, then

  Area of triangle $= \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$
- Three points $(x_1, y_1), (x_2, y_2), (x_3, y_3)$ are collinear, if

  $$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$
- Equation of a line joining the points (x_1, y_1) and (x_2, y_2) is:

  $$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$
- **Minor of an element** a_{ij} of a determinant is the determinant obtained by deleting ith row and jth column in which element a_{ij} lies. It is denoted by M_{ij}.

  The minor of an element of a determinant of order n $(n \geq 2)$ is a determinant of order $n-1$.
- **Cofactor of element** a_{ij} of a square matrix A is $C_{ij} = (-1)^{i+j} \times M_{ij}$, where M_{ij} = minor of element a_{ij}.
- The transposes of the matrix formed by the cofactors of corresponding elements of a square matrix A is called **adjoint matrix** A which is denoted by adj A.
- **Properties of Adjoint of a Matrix**

  If A and B are square matrices of order n, then

  (i) $A(\text{adj } A) = |A|\, I_n = (\text{adj } A)\, A$

  (ii) $\text{adj}(A^T) = (\text{adj } A)^T$

  (iii) $\text{adj}(AB) = \text{adj}(B)\, \text{adj}(A)$

  (iv) $\text{adj}(kA) = k^{n-1}(\text{adj } A), k \in R$

  (v) $\text{adj}(A^m) = (\text{adj } A)^m, m \in N$
- If the determinant value of a square matrix is zero *i.e.*, $|A| = 0$, then that matrix is called **singular matrix**.
- If the determinant value of a square matrix is non-zero *i.e.*, $|A| \neq 0$, then it is called **non-singular matrix**.
- If A is a non-singular square matrix of order n, then

  (i) $|\text{adj } A| = |A|^{n-1}$

  (ii) $\text{adj}(\text{adj } A) = |A|^{n-2} A$

  (iii) $|\text{adj}(\text{adj } A)| = |A|^{(n-1)^2}$

  (iv) If A is a square matrix of order n such that $|A| \neq 0$, then

  $$\frac{A(\text{adj } A)}{|A|} = \frac{(\text{adj } A)\, A}{|A|} = I$$
- The square matrix A is invertible if it is non-singular.

  i.e., $A^{-1} = \frac{1}{|A|}(\text{adj } A)$ and $(A^{-1})^{-1} = A$
- **Properties of Inverse of a Square Matrix**

  (i) $(A^{-1})^{-1} = A$

  (ii) $(A^T)^{-1} = (A^{-1})^T$

  (iii) $|A^{-1}| = |A|^{-1}$

  (iv) $(A^k)^{-1} = (A^{-1})^k, k \in N$

  (v) If A and B are two non-singular matrices of same order, then

  $$(AB)^{-1} = B^{-1}A^{-1}$$
- If $a_1x + b_1y + c_1z = d_1$, $a_2x + b_2y + c_2z = d_2$, $a_3x + b_3y + c_3z = d_3$, then these equations can be written in the matrix form as $AX = B$, where

  $$A = \begin{bmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_3 & c_3 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} d_1 \\ d_2 \\ d_3 \end{bmatrix}$$
- Unique solution of equations $AX = B$ is given by $X = A^{-1}B$ where $|A| \neq 0$.
- A system of equation is consistent or inconsistent according to its solution exists or not.
- For a square matrix A in matrix equations $AX = B$:

  (i) If $|A| \neq 0$, then the system of equations is consistent and has a unique solution given by $X = A^{-1}B$.

  (ii) If $|A| = 0$ and $(\text{adj } A)B = O$, then the system of equations is consistent and has infinitely many solutions.

  (iii) If $|A| = 0$ and $(\text{adj } A)B \neq O$, then the system of equations is inconsistent *i.e.*, having no solution.

Practice Exercise

Multiple Choice Questions

Q 1. The value of the determinant $\begin{vmatrix} 6 & 0 & -1 \\ 2 & 1 & 4 \\ 1 & 1 & 3 \end{vmatrix}$ is: *(CBSE 2023)*

a. 10 b. 8 c. 7 d. -7

Q 2. If $\begin{vmatrix} \alpha & 3 & 4 \\ 1 & 2 & 1 \\ 1 & 4 & 1 \end{vmatrix} = 0$, then the value of α is: *(CBSE 2023)*

a. 1 b. 2 c. 3 d. 4

Q 3. The value of the determinant $\begin{vmatrix} 2 & 7 & 1 \\ 1 & 1 & 1 \\ 10 & 8 & 1 \end{vmatrix}$ is: *(CBSE 2023)*

a. 47 b. -79 c. 49 d. -51

Q 4. The value of $\begin{vmatrix} x+y & y+z & z+x \\ z & x & y \\ 1 & 1 & 1 \end{vmatrix}$ is: *(CBSE 2023)*

a. 0
b. 1
c. $x+y+z$
d. $2(x+y+z)$

Q 5. The value of $|A|$, if $A = \begin{bmatrix} 0 & 2x-1 & \sqrt{x} \\ 1-2x & 0 & 2\sqrt{x} \\ -\sqrt{x} & -2\sqrt{x} & 0 \end{bmatrix}$, where $x \in R^+$, is: *(CBSE SQP 2023-24)*

a. $(2x+1)^2$
b. 0
c. $(2x+1)^3$
d. None of these

Q 6. If $\begin{vmatrix} 2 & 4 \\ 5 & 1 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ 6 & x \end{vmatrix}$, then the possible value(s) of 'x' is/are: *(CBSE SQP 2022-23)*

a. 3 b. $\sqrt{3}$ c. $-\sqrt{3}$ d. $\sqrt{3}, -\sqrt{3}$

Q 7. Solution set of the equation $\begin{vmatrix} x & -6 & -1 \\ 2 & -3x & x-3 \\ -3 & 2x & x+2 \end{vmatrix} = 0$ is:

a. $\{2, 0, 1\}$
b. $\{2, -3, 1\}$
c. $\{2, 1, 5\}$
d. $\{-3, 1, 5\}$

Q 8. If $C = 2\cos\theta$, then the value of the determinant $\Delta = \begin{vmatrix} C & 1 & 0 \\ 1 & C & 1 \\ 0 & 1 & C \end{vmatrix}$ is:

a. $\dfrac{\sin 4\theta}{\sin\theta}$
b. $\dfrac{2\sin^2 2\theta}{\sin\theta}$
c. $4\cos^2\theta(2\cos\theta - 1)$
d. None of these

Q 9. If $\begin{vmatrix} \alpha & -\beta & 0 \\ 0 & \alpha & \beta \\ \beta & 0 & \alpha \end{vmatrix} = 0$, then:

a. $\dfrac{\alpha}{\beta}$ is one of the cube roots of unity
b. α is one of the cube roots of unity
c. β is one of the cube roots of unity
d. None of the above

Q 10. If A is a square matrix such that $A^2 = A$, then $|A|$ equals:

a. 0 or 1
b. -2 or 2
c. -3 or 3
d. None of these

Q 11. If A is 3×3 matrix such that $|A| = 8$, then $|3A|$ equals: *(CBSE 2020)*

a. 8 b. 24 c. 72 d. 216

Q 12. If A and B are any 2×2 matrices, then $\det(A+B) = 0$ implies:

a. $\det A = 0$ and $\det B = 0$
b. $\det A + \det B \neq 0$
c. $\det A = 0$ or $\det B = 0$
d. None of these

Q 13. The value of

$$\Delta = \begin{vmatrix} \cos(\alpha+\beta) & -\sin(\alpha+\beta) & \cos 2\beta \\ \sin\alpha & \cos\alpha & \sin\beta \\ -\cos\alpha & \sin\alpha & \cos\beta \end{vmatrix}$$

is independent of:

a. α
b. β
c. α, β
d. None of these

Q 14. If A and B are square matrices each of order 3 and $|A| = 5, |B| = 3$, then the value of $|3AB|$ is: *(CBSE 2020)*

a. 27
b. 15
c. 405
d. 42

Q 15. Let $A = \begin{bmatrix} 1 & \sin\alpha & 1 \\ -\sin\alpha & 1 & \sin\alpha \\ -1 & -\sin\alpha & 1 \end{bmatrix}$, where $0 \le \alpha \le 2\pi$, then: *(CBSE SQP 2021 Term-1)*

a. $|A| = 0$
b. $|A| \in (2, \infty)$
c. $|A| \in (2, 4)$
d. $|A| \in [2, 4]$

Q 16. If $x = -4$ is a root of $\begin{vmatrix} x & 2 & 3 \\ 1 & x & 1 \\ 3 & 2 & x \end{vmatrix} = 0$, then the sum of the other two roots is: *(CBSE 2021 Term-1)*

a. 4
b. -3
c. 2
d. 5

Q 17. If for the matrix $A = \begin{bmatrix} \alpha & -2 \\ -2 & \alpha \end{bmatrix}$, $|A^3| = 125$, then the value of α is: *(CBSE 2021 Term-1)*

a. ± 3
b. -3
c. ± 1
d. 1

Q 18. If A is a matrix of order 3×3, then number of minors in determinant of A are: *(NCERT EXEMPLAR)*

a. 3 b. 6

c. 9 d. 18

Q 19. Find the cofactors of a_{21} and a_{31} of the matrix

$$A = [a_{ij}] = \begin{bmatrix} 1 & 3 & -2 \\ 4 & -5 & 6 \\ 3 & 5 & 2 \end{bmatrix}.$$

a. $-16, 8$ b. $-16, -8$

c. $16, 8$ d. $16, -8$

Q 20. The cofactors of all the elements $\begin{bmatrix} 1 & -2 \\ 4 & 3 \end{bmatrix}$ are: *(CBSE 2020)*

a. $3, -4, 2, 1$ b. $-3, 4, -2, 1$

c. $-3, 4, -2, -1$ d. None of these

Q 21. If A_{ij} denotes the cofactor of the element a_{ij} of the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then the value of $a_{11}A_{31} + a_{12}A_{32} + a_{13}A_{33}$ is:

a. 0 b. 5

c. 10 d. -5

Q 22. Given that $A = [a_{ij}]$ is square matrix of order 3×3 and $|A| = -7$, then the value of $\sum_{i=1}^{2} a_{i2}A_{i2}$, where A_{ij} denotes the cofactor of element a_{ij}, is: *(CBSE SQP 2021 Term-1)*

a. 7 b. -7

c. 0 d. 49

Q 23. If C_{ij} denotes the cofactor of element p_{ij} of the matrix $P = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & 2 & 4 \end{bmatrix}$, then the value of $C_{31} \cdot C_{23}$ is: *(CBSE 2021 Term-1)*

a. 5 b. 24

c. -24 d. -5

Q 24. Let matrix $X = [x_{ij}]$ is given by $X = \begin{bmatrix} 1 & -1 & 2 \\ 3 & 4 & -5 \\ 2 & -1 & 3 \end{bmatrix}$. Then the matrix $Y = [m_{ij}]$, where m_{ij} = Minor of x_{ij}, is: *(CBSE 2021 Term-1)*

a. $\begin{bmatrix} 7 & -5 & -3 \\ 19 & 1 & -11 \\ -11 & 1 & 7 \end{bmatrix}$ b. $\begin{bmatrix} 7 & -19 & -11 \\ 5 & -1 & -1 \\ 3 & 11 & 7 \end{bmatrix}$

c. $\begin{bmatrix} 7 & 19 & -11 \\ -3 & 11 & 7 \\ -5 & -1 & -1 \end{bmatrix}$ d. $\begin{bmatrix} 7 & 19 & -11 \\ -1 & -1 & 1 \\ -3 & -11 & 7 \end{bmatrix}$

Q 25. If the area of a triangle with vertices $(2, -6)$, $(5, 4)$ and $(k, 4)$ is 35 sq. units, then k is: *(CBSE 2023)*

a. 12 b. -2

c. $-12, -2$ d. $12, -2$

Q 26. If the area of a triangle with vertices $(-3, 0)$, $(3, 0)$ and $(0, k)$ is 9 sq. units, then the value(s) of k will be: *(CBSE SQP 2023-24)*

a. 9 b. ± 3 c. -9 d. 6

Q 27. Three points $P(2x, x+3)$, $Q(0, x)$ and $R(x+3, x+6)$ are collinear, then x is equal to: *(CBSE 2021 Term-1)*

a. 0 b. 2 c. 3 d. 1

Q 28. Value of k, for which $A = \begin{bmatrix} k & 8 \\ 4 & 2k \end{bmatrix}$ is a singular matrix, is: *(CBSE SQP 2021 Term-1)*

a. 4 b. -4

c. ± 4 d. 0

Q 29. Given that A is a non-singular matrix of order 3 such that $A^2 = 2A$, then value of $|2A|$ is: *(CBSE SQP 2021 Term-1)*

a. 4 b. 8

c. 64 d. 16

Q 30. If A and B are invertible square matrices of the same order, then which of the following is not correct? *(CBSE SQP 2023-24)*

a. $\text{adj } A = |A| \cdot A^{-1}$ b. $\det(A)^{-1} = [\det(A)]^{-1}$

c. $(AB)^{-1} = B^{-1}A^{-1}$ d. $(A+B)^{-1} = B^{-1} + A^{-1}$

Q 31. Given that A is a square matrix of order 3 and $|A| = -2$, then $|\text{adj}(2A)|$ is equal to: *(CBSE SQP 2023-24)*

a. -2^6 b. 4

c. -2^8 d. 2^8

Q 32. For matrix $A = \begin{bmatrix} 2 & 5 \\ -11 & 7 \end{bmatrix}$, $(\text{adj } A)'$ is equal to: *(CBSE SQP 2021 Term-1)*

a. $\begin{bmatrix} -2 & -5 \\ 11 & -7 \end{bmatrix}$ b. $\begin{bmatrix} 7 & 5 \\ 11 & 2 \end{bmatrix}$

c. $\begin{bmatrix} 7 & 11 \\ -5 & 2 \end{bmatrix}$ d. $\begin{bmatrix} 7 & -5 \\ 11 & 1 \end{bmatrix}$

Q 33. If A is a square matrix of order 3 and $|A| = -5$, then $|\text{adj } A|$ is: *(CBSE 2021 Term-1)*

a. 125 b. -25

c. 25 d. ± 25

Q 34. If $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$, then the value of $|A||\text{adj } A|$ is:

a. a^3 b. a^6

c. a^9 d. a^{27}

Q 35. If A is a square matrix of order 3 and $|A| = 5$, then $|\text{adj } A| =$ *(CBSE SQP 2022-23)*

a. 5 b. 25

c. 125 d. $\frac{1}{5}$

Q 36. If A is a square matrix of order 3, such that $A(\text{adj } A) = 10I$, then $|\text{adj } A|$ is equal to: *(CBSE 2020)*

a. 1 b. 10

c. 100 d. 101

Q 37. If A is a 3×3 non-singular matrix, then $|A^{-1}\,\text{adj}(A)|$ is:

a. $|A|$ b. 1

c. $|A|^2$ d. $|A|^{-1}$

Q 38. If $A = \begin{bmatrix} 2 & \lambda & -3 \\ 0 & 2 & 5 \\ 1 & 1 & 3 \end{bmatrix}$, then A^{-1} exists if:

(NCERT EXEMPLAR)

a. $\lambda = 2$ b. $\lambda \neq 2$

c. $\lambda \neq -2$ d. None of these

Q 39. The multiplicative inverse of $A = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$ is:

a. $\begin{bmatrix} -\cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$ b. $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$

c. $\begin{bmatrix} -\cos\theta & -\sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$ d. $\begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & -\cos\theta \end{bmatrix}$

Q 40. $A = \begin{bmatrix} 3 & 1 \\ -1 & 2 \end{bmatrix}$, then $14A^{-1}$ is given by:

(CBSE SQP 2021 Term-1)

a. $14\begin{bmatrix} 2 & -1 \\ 1 & 3 \end{bmatrix}$ b. $\begin{bmatrix} 4 & -2 \\ 2 & 6 \end{bmatrix}$

c. $2\begin{bmatrix} 2 & -1 \\ 1 & -3 \end{bmatrix}$ d. $2\begin{bmatrix} -3 & -1 \\ 1 & -2 \end{bmatrix}$

Q 41. If $A = \begin{bmatrix} 2 & 3 \\ 1 & -4 \end{bmatrix}$ and $B = \begin{bmatrix} 1 & -2 \\ -1 & 3 \end{bmatrix}$, then find $(AB)^{-1}$.

a. $\frac{1}{11}\begin{bmatrix} 14 & 5 \\ 5 & 1 \end{bmatrix}$ b. $\frac{1}{11}\begin{bmatrix} 14 & -5 \\ -5 & 1 \end{bmatrix}$

c. $\frac{1}{11}\begin{bmatrix} 1 & 5 \\ 5 & 14 \end{bmatrix}$ d. $\frac{1}{11}\begin{bmatrix} 1 & -5 \\ -5 & 14 \end{bmatrix}$

Q 42. The multiplicative inverse of the matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$ is:

a. $\begin{bmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ b. $\begin{bmatrix} 0 & 1 & -1 \\ 0 & -1 & 0 \\ -1 & 0 & 0 \end{bmatrix}$

c. $\begin{bmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 1 & 0 & -1 \end{bmatrix}$ d. $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

Q 43. If $A = \begin{bmatrix} -4 & -3 & -3 \\ 1 & 0 & 1 \\ 4 & 4 & 3 \end{bmatrix}$, then adj $A =$

a. A^{-1} b. A^T

c. A d. Both a. and c.

Q 44. If $A = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$, then $\dfrac{A^2 - 3I}{2} =$

a. A^{-1} b. $2A$ c. $2A^{-1}$ d. $\frac{3}{2}A^{-1}$

Q 45. The inverse of the matrix $X = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ is:

(CBSE 2021 Term-1)

a. $24\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$ b. $\frac{1}{24}\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

c. $\frac{1}{24}\begin{bmatrix} 2 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 4 \end{bmatrix}$ d. $\begin{bmatrix} 1/2 & 0 & 0 \\ 0 & 1/3 & 0 \\ 0 & 0 & 1/4 \end{bmatrix}$

Q 46. The system of linear equations $x + y + z = 2$, $2x + y - z = 3$, $3x + 2y + kz = 4$ has a unique solution, if k is not equal to:

a. 4 b. -4 c. 0 d. 3

Assertion & Reason Type Questions

Directions (Q. Nos. 47-55): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 47. Assertion (A): If a, b, c are distinct and x, y, z are not all zero, given that $ax + by + cz = 0$, $bx + cy + az = 0$, $cx + ay + bz = 0$, then $a + b + c \neq 0$.

Reason (R): $a^2 + b^2 + c^2 > ab + bc + ca$, if a, b and c are distinct.

Q 48. Assertion (A): The determinant of a matrix $\begin{bmatrix} 0 & p-q & p-r \\ q-p & 0 & q-r \\ r-p & r-q & 0 \end{bmatrix}$ is zero.

Reason (R): The determinant of a skew-symmetric matrix of odd order is zero.

Q 49. Assertion (A): If A is skew-symmetric matrix of order 3, then its determinant should be zero.

Reason (R): If A is a square matrix then $\det A = \det A' = \det(-A')$.

Q 50. Assertion (A): $\Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$ where, A_{ij} is cofactor of a_{ij}.

Reason (R): $\Delta =$ Sum of the products of elements of any row (or column) with their corresponding cofactors.

Q 51. Let A be a 2×2 matrix.

Assertion (A): adj (adj A) $= A$

Reason (R): $|\text{adj } A| = |A|$

Q 52. Assertion (A): If $A_r = \begin{bmatrix} r & r-1 \\ r-1 & r \end{bmatrix}$, where r is a natural number, then

$|A_1| + |A_2| + \ldots + |A_{2006}| = (2006)^2$.

Reason (R): If A is a matrix of order 3 and $|A| = 2$, then $|\text{adj } A| = 2^2$.

Q 53. Assertion (A): The matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 8 \end{bmatrix}$ is singular.

Reason (R): A square matrix A is said to be singular, if $|A| = 0$.

Q 54. Assertion (A): There are only finitely many 2×2 matrices which commute with the matrix $\begin{bmatrix} 1 & 2 \\ -1 & -1 \end{bmatrix}$.

Reason (R): If A is non-singular, then it commutes with I, adj A and A^{-1}.

Q 55. Assertion (A): The system of equations $2x - y = -2$; $3x + 4y = 3$ has unique solution and $x = -\frac{5}{11}$ and $y = \frac{12}{11}$.

Reason (R): The system of equations $AX = B$ has a unique solution, if $|A| \neq 0$.

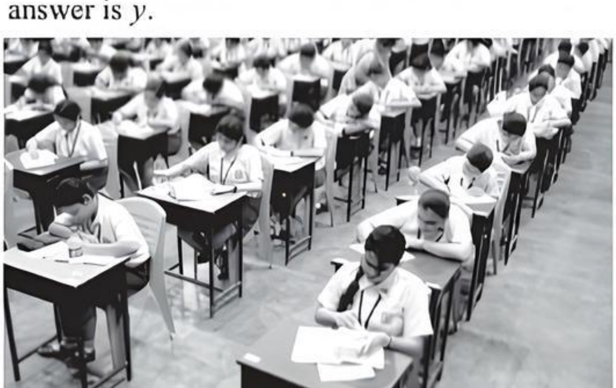
Answers

1. (d)	2. (d)	3. (a)	4. (a)	5. (b)	6. (d)	7. (b)	8. (a)	9. (a)	10. (a)
11. (d)	12. (b)	13. (a)	14. (c)	15. (d)	16. (a)	17. (a)	18. (c)	19. (a)	20. (a)
21. (a)	22. (b)	23. (a)	24. (d)	25. (d)	26. (b)	27. (d)	28. (c)	29. (c)	30. (d)
31. (d)	32. (c)	33. (c)	34. (c)	35. (b)	36. (b)	37. (a)	38. (d)	39. (b)	40. (b)
41. (a)	42. (d)	43. (d)	44. (a)	45. (d)	46. (c)	47. (d)	48. (a)	49. (c)	50. (a)
51. (b)	52. (b)	53. (a)	54. (d)	55. (a)					

Case Study Based Questions

Case Study 1

Yash scored 40 marks in a test, getting 3 marks for each right answer and losing 1 mark for each wrong answer. Had 4 marks been awarded for each correct answer and 2 marks been deducted for each incorrect answer, then Yash would have scored 50 marks. Let number of right answer is x and number of wrong answer is y.



Based on the above information, solve the following questions:

Q 1. The equation in terms of x and y are:

a. $3x - y = 40$, $2x - y = 25$

b. $x - 3y = 40$, $x - 2y = 25$

c. $3x + y = 40$, $2x + y = 25$

d. $x + 3y = 40$, $x + 2y = 25$

Q 2. Which of the following matrix equations represent the above information?

a. $\begin{bmatrix} 1 & 3 \\ 1 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 25 \end{bmatrix}$

b. $\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 25 \end{bmatrix}$

c. $\begin{bmatrix} 1 & -3 \\ 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 25 \end{bmatrix}$

d. $\begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 25 \end{bmatrix}$

Q 3. Using matrix method, find the number of right answer given by Yash.

a. 5 b. 15

c. 20 d. 40

Q 4. The number of wrong answers given by Yash are:

a. 15 b. 20

c. 5 d. 25

Q 5. How many questions were there in the test?

a. 20 b. 25 c. 35 d. 40

Solutions

1. Let number of right answers $= x$

and number of wrong answers $= y$

$\therefore$ Total number of questions $= x + y$

In First Case:

Marks awarded for x right answers $= 3x$

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Negative (–) sign is used to indicate marks lost for wrong answer.

Marks lost for y wrong answers $= y \times (-1) = -y$

Then, $3x - y = 40$ (by given condition) ...(1)

In Second Case:

Marks awarded for x right answers $= 4x$

Marks lost for y wrong answers $= y \times (-2) = -2y$

Then, $4x - 2y = 50$ (by given condition)

or $2x - y = 25$ (divide by 2 on both sides) ...(2)

So, option (a) is correct.

2. Given equations can be written in the form of matrix $AX = B$ as

$$\begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 40 \\ 25 \end{bmatrix}$$

where, $A = \begin{bmatrix} 3 & -1 \\ 2 & -1 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 40 \\ 25 \end{bmatrix}$

So, option (d) is correct.

Sol. (Q.Nos. 3-5): $|A| = \begin{vmatrix} 3 & -1 \\ 2 & -1 \end{vmatrix} = -3 + 2 = -1 \neq 0$

If A_{ij} is the cofactor of the elements a_{ij}, then

$A_{11} = (-1)^{1+1}(-1) = -1$, $A_{12} = (-1)^{1+2}(2) = -2$

$A_{21} = (-1)^{2+1}(-1) = 1$ and $A_{22} = (-1)^{2+2}(3) = 3$

$$\therefore \text{ adj } A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^T = \begin{bmatrix} -1 & -2 \\ 1 & 3 \end{bmatrix}^T = \begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|}\text{adj } A = \frac{1}{-1}\begin{bmatrix} -1 & 1 \\ -2 & 3 \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}$$

Now, we have

$X = A^{-1}B$

$$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 2 & -3 \end{bmatrix}\begin{bmatrix} 40 \\ 25 \end{bmatrix} = \begin{bmatrix} 40 - 25 \\ 80 - 75 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 15 \\ 5 \end{bmatrix}$$

$\Rightarrow$ $x = 15$ and $y = 5$

3. The number of right answer is x *i.e.*, 15.
   So, option (b) is correct.
4. The number of wrong answer is y *i.e.*, 5.
   So, option (c) is correct.
5. Total number of questions $= x + y = 15 + 5 = 20$
   So, option (a) is correct.

Case Study 2

Anika wants to donate a rectangular plot of land for an orphanage. When she was asked to give dimensions of the plot, she told that the area of a rectangle gets reduced by 9 sq. units, if its length is reduced by 5 units and breadth is increased by 3 units, but if increase the length by 3 units and breadth by 2 units, the area increase by 67 sq. units.

Let x and y be the length and breadth of a rectangular plot.

Based on the given information, solve the following questions:

Q 1. The equations in terms of x and y are:

a. $3x - 5y = 6$, $2x + 3y = 61$

b. $5x - 3y = 6$, $3x + 2y = 61$

c. $3x - 5y = 61$, $2x + 3y = 6$

d. $5x - 3y = 61$, $3x + 2y = 6$

Q 2. Which of the following matrix equations represent the above information?

a. $\begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 61 \end{bmatrix}$

b. $\begin{bmatrix} 3 & -5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 61 \end{bmatrix}$

c. $\begin{bmatrix} 3 & -5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 61 \\ 6 \end{bmatrix}$

d. $\begin{bmatrix} 5 & -3 \\ 3 & 2 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 61 \\ 6 \end{bmatrix}$

Q 3. Using matrix method, find the length and breadth of the rectangular plot.

a. 26 units and 8 units
b. 9 units and 15 units
c. 17 units and 9 units
d. 8 units and 16 units

Q 4. How much is the perimeter of rectangular plot?

a. 25 units
b. 32 units
c. 45 units
d. 52 units

Q 5. How much is the area of rectangular plot?

a. 153 sq. units
b. 170 sq. units
c. 90 sq. units
d. 52 sq. units

Solutions

1. Let x and y be the length and breadth of a rectangular plot.

In First Case:

Area is reduced by 9 sq. units when length $= (x - 5)$ units

and breadth $= (y + 3)$ units

$\because$ Area of rectangular plot = length $\times$ breadth $= xy$

According to given condition,

$xy - (x - 5)(y + 3) = 9$

$\Rightarrow xy - (xy - 5y + 3x - 15) = 9$

$\Rightarrow xy - xy + 5y - 3x + 15 = 9$

$\Rightarrow -3x + 5y = -6 \Rightarrow 3x - 5y = 6$ ...(1)

In Second Case:

Area is increased by 67 sq. units when length $= (x + 3)$ units

and breadth $= (y + 2)$ units

According to given condition,

$(x + 3)(y + 2) - xy = 67$

$\Rightarrow xy + 3y + 2x + 6 - xy = 67$

$\Rightarrow 2x + 3y = 61$ ...(2)

So, option (a) is correct.

2. Given equation can be written in the form of matrix $AX = B$ as

$$\begin{bmatrix} 3 & -5 \\ 2 & 3 \end{bmatrix}\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 6 \\ 61 \end{bmatrix}$$

where, $A = \begin{bmatrix} 3 & -5 \\ 2 & 3 \end{bmatrix}$, $X = \begin{bmatrix} x \\ y \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 61 \end{bmatrix}$

So, option (b) is correct.

Sol. (Q.Nos. 3-5): $|A| = \begin{vmatrix} 3 & -5 \\ 2 & 3 \end{vmatrix} = 9 + 10 = 19$

If A_{ij} is the cofactor of the element a_{ij}, then

$A_{11} = (-1)^{1+1}(3) = 3,\ A_{12} = (-1)^{1+2}(2) = -2$

$A_{21} = (-1)^{2+1}(-5) = 5$ and $A_{22} = (-1)^{2+2}(3) = 3$

$\therefore\ \text{adj } A = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix}^T = \begin{bmatrix} 3 & -2 \\ 5 & 3 \end{bmatrix}^T = \begin{bmatrix} 3 & 5 \\ -2 & 3 \end{bmatrix}$

$\therefore\ A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{19}\begin{bmatrix} 3 & 5 \\ -2 & 3 \end{bmatrix}$

Now, we have $X = A^{-1}B$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 3 & 5 \\ -2 & 3 \end{bmatrix}\begin{bmatrix} 6 \\ 61 \end{bmatrix} = \frac{1}{19}\begin{bmatrix} 18 + 305 \\ -12 + 183 \end{bmatrix}$

$\Rightarrow \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 323/19 \\ 171/19 \end{bmatrix} = \begin{bmatrix} 17 \\ 9 \end{bmatrix}$

$\Rightarrow\ x = 17$ and $y = 9$

3. The length of the rectangular plot = x units = 17 units and the breadth of the rectangular plot

= y units = 9 units

So, option (c) is correct.

4. The perimeter of the rectangular plot

$= 2(x + y) = 2(17 + 9)$

$= 2 \times 26 = 52$ units

So, option (d) is correct.

5. The area of the rectangular plot

$= x \times y = 17 \times 9 = 153$ sq. units

So, option (a) is correct.

Case Study 3

Minor of an element a_{ij} of a determinant is the determinant obtained by deleting its ith row and jth column in which element a_{ij} lies and is denoted by M_{ij}. Cofactor of an element a_{ij}, denoted by A_{ij}, is defined by $A_{ij} = (-1)^{i+j} M_{ij}$, where M_{ij} is minor of a_{ij}.

Also, the determinant of a square matrix A is the sum of the products of the elements of any row (or column) with their corresponding cofactors. For example, if $A = [a_{ij}]_{3\times 3}$, then $|A| = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

Based on the above information, solve the following questions:

Q 1. Find the sum of the cofactors of all the elements of $\begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$.

a. 2 b. −2
c. 4 d. 1

Q 2. Find the minor of a_{21} of $\begin{vmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{vmatrix}$.

a. 3 b. −3
c. 39 d. −39

Q 3. In the determinant $\begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, find the value of $a_{32} \cdot A_{32}$.

a. 27 b. −110
c. 110 d. −27

Q 4. If $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$, then write the minor of a_{23}.

a. −10 b. −7
c. 10 d. 7

Q 5. If $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$, then find the value of $|\Delta|$.

a. 26 b. 28
c. 72 d. 46

Solutions

1. Let $\Delta = \begin{vmatrix} 1 & -2 \\ 4 & 3 \end{vmatrix}$

Cofactor of $1 = 3$, cofactor of $-2 = -4$

Cofactor of $4 = 2$, Cofactor of $3 = 1$

$\therefore$ Required sum $= 3 - 4 + 2 + 1 = 2$

So, option (a) is correct.

2. Let $\Delta = \begin{vmatrix} 5 & 6 & -3 \\ -4 & 3 & 2 \\ -4 & -7 & 3 \end{vmatrix}$

Minor of $a_{21} = \begin{vmatrix} 6 & -3 \\ -7 & 3 \end{vmatrix} = 18 - 21 = -3$

So, option (b) is correct.

3. Let $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

Clearly, $a_{32} = 5$

and A_{32} = cofactor of a_{32} in $\Delta = (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 6 & 4 \end{vmatrix}$

$= (-1)(8 - 30) = 22$

$\therefore\ a_{32} \cdot A_{32} = 5 \times 22 = 110$

So, option (c) is correct.

4. Here, $\Delta = \begin{vmatrix} 5 & 3 & 8 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{vmatrix}$

$\therefore$ Minor of $a_{23} = \begin{vmatrix} 5 & 3 \\ 1 & 2 \end{vmatrix} = 10 - 3 = 7$

So, option (d) is correct.

5. Here, $\Delta = \begin{vmatrix} 2 & -3 & 5 \\ 6 & 0 & 4 \\ 1 & 5 & -7 \end{vmatrix}$

$A_{11} = (-1)^{1+1}\begin{vmatrix} 0 & 4 \\ 5 & -7 \end{vmatrix} = 1(0 - 20) = -20,$

$A_{12} = (-1)^{1+2}\begin{vmatrix} 6 & 4 \\ 1 & -7 \end{vmatrix} = -1(-42-4) = 46.$

$A_{13} = (-1)^{1+3}\begin{vmatrix} 6 & 0 \\ 1 & 5 \end{vmatrix} = 1(30-0) = 30$

$\therefore \quad \Delta = a_{11}A_{11} + a_{12}A_{12} + a_{13}A_{13}$

$= 2(-20) - 3(46) + 5(30) = -28$

$\Rightarrow |\Delta| = 28$

So, option (b) is correct.

Case Study 4

If there is a statement involving the natural number n such that

(i) The statement is true for $n = 1$

(ii) When the statement is true for $n = k$ (where k is some positive integer), then the statement is also true for $n = k+1$

Then, the statement is true for all natural numbers n. Also, if A is a square matrix of order n, then A^2 is defined as AA. In general $A^m = AA \dots A$ (m times), where m is any positive integer.

Based on the above information, solve the following questions:

Q 1. If $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, then for any positive integer n:

a. $A^n = \begin{bmatrix} 3n & -4n \\ n & -n \end{bmatrix}$ b. $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

c. $A^n = \begin{bmatrix} 3n & -8n \\ 1 & -n \end{bmatrix}$ d. $A^n = \begin{bmatrix} 1+3n & -4n \\ n & 1-3n \end{bmatrix}$

Q 2. If $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$, then $|A^n|$, where $n \in N$, is equal to:

a. 2^n b. 3^n

c. n d. 1

Q 3. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which of the following holds for all natural number $n \geq 1$?

a. $A^n = nA - (n-1)I$ b. $A^n = 2^{n-1}A - (n-1)I$

c. $A^n = nA + (n-1)I$ d. $A^n = 2^{n-1}A + (n-1)I$

Q 4. Let $A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$ and $A^n = [a_{ij}]_{3\times 3}$ for some positive integer n, then the cofactor of a_{13} is:

a. a^n b. $-a^n$

c. $2a^n$ d. 0

Q 5. If A is a square matrix such that $|A| = 2$, then for any positive integer n, $|A^n|$ is equal to:

a. 0 b. $2n$

c. 2^n d. n^2

Solutions

1. We have, $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$

$\therefore \quad A^2 = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}\begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix} = \begin{bmatrix} 5 & -8 \\ 2 & -3 \end{bmatrix}$, which can be obtained $A^n = \begin{bmatrix} 1+2n & -4n \\ n & 1-2n \end{bmatrix}$

So, option (b) is correct.

2. We have, $A = \begin{bmatrix} 1 & 2 \\ 0 & 1 \end{bmatrix}$

$\therefore \quad |A| = \begin{vmatrix} 1 & 2 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$

Also, $|A^n| = |A \cdot A \dots A \text{ (}n\text{ times)}| = |A|^n = 1^n = 1$

So, option (d) is correct.

3. For $n = 1$, all options are true.

$A^2 = A \cdot A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$

and $\quad A^3 = A^2 \cdot A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$

Putting $n = 3$, in option (a) we get $A^3 = 3A - 2I$

$= 3\begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}$

$= \begin{bmatrix} 3 & 0 \\ 3 & 3 \end{bmatrix} - \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ 3 & 1 \end{bmatrix}$, which is true.

All other options are different from $A^3 = 3A - 2I$ for $n = 3$

So, option (a) is correct.

4. We have $\quad A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

$\therefore \quad A^2 = A \cdot A = \begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}\begin{bmatrix} a & 0 & 0 \\ 0 & a & 0 \\ 0 & 0 & a \end{bmatrix}$

$= \begin{bmatrix} a^2 & 0 & 0 \\ 0 & a^2 & 0 \\ 0 & 0 & a^2 \end{bmatrix}$

Similarly, $\quad A^n = \begin{bmatrix} a^n & 0 & 0 \\ 0 & a^n & 0 \\ 0 & 0 & a^n \end{bmatrix}$

Now, cofactor of $a_{13} = (-1)^{1+3}\begin{vmatrix} 0 & a^n \\ 0 & 0 \end{vmatrix} = 0$

So, option (d) is correct.

5. We have, $\quad |A| = 2$

and $\quad |A^n| = |A \cdot A \dots A \text{ (}n\text{-times)}|$

$= |A||A| \dots |A| \text{ (}n\text{-times)} = |A|^n = 2^n$

So, option (c) is correct.

Case Study 5

Gautam buys 5 pens, 3 bags and 1 instrument box and pays a sum of ₹ 160. From the same shop, Vikram buys 2 pens, 1 bag and 3 instruments boxes and pays a sum of ₹ 190. Also Ankur buys 1 pen, 2 bags and 4 instruments boxes and pays a sum of ₹ 250.

Based on the above information, solve the following questions: (CBSE 2023)

Q 1. Convert the given above situation into a matrix equation of the form $AX = B$.

Q 2. Find $|A|$.

Q 3. Find A^{-1}.

Or

Determine $P = A^2 - 5A$.

Solutions

1. Let, cost of 1 pen = ₹ x
cost of 1 bag = ₹ y
and cost of 1 instrument box = ₹ z
According to the question, we have

$$5x + 3y + z = 160$$
$$2x + y + 3z = 190$$
and $x + 2y + 4z = 250$

This system of equation can be written as $AX = B$

where, $A = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}, B = \begin{bmatrix} 160 \\ 190 \\ 250 \end{bmatrix}$ and $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$

2. Now $|A| = 5(4-6) - 3(8-3) + 1(4-1)$
$= -10 - 3(5) + 3 = -10 - 15 + 3$
$= -22$

3. $\because$ $|A| \neq 0$, so A^{-1} exists.

$$\text{Here, adj } A = \begin{bmatrix} -2 & -5 & 3 \\ -10 & 19 & -7 \\ 8 & -13 & -1 \end{bmatrix}^T = \begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{-22}\begin{bmatrix} -2 & -10 & 8 \\ -5 & 19 & -13 \\ 3 & -7 & -1 \end{bmatrix}$$

Or

Given $P = A^2 - 5A$

$$\therefore P = \begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix} - 5\begin{bmatrix} 5 & 3 & 1 \\ 2 & 1 & 3 \\ 1 & 2 & 4 \end{bmatrix}$$

$$= \begin{bmatrix} 25+6+1 & 15+3+2 & 5+9+4 \\ 10+2+3 & 6+1+6 & 2+3+12 \\ 5+4+4 & 3+2+8 & 1+6+16 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix}$$

$$= \begin{bmatrix} 32 & 20 & 18 \\ 15 & 13 & 17 \\ 13 & 13 & 23 \end{bmatrix} - \begin{bmatrix} 25 & 15 & 5 \\ 10 & 5 & 15 \\ 5 & 10 & 20 \end{bmatrix} = \begin{bmatrix} 7 & 5 & 13 \\ 5 & 8 & 2 \\ 8 & 3 & 3 \end{bmatrix}$$

Case Study 6

Three shopkeepers Sanjeev, Rohit and Deepak are using polythene bags, handmade bags (prepared by prisoners) and newspaper's envelope as carry bags. It is found that the shopkeepers Sanjeev, Rohit and Deepak are using (20, 30, 40), (30, 40, 20) and (40, 20, 30) polythene bags, handmade bags and newspaper's envelopes respectively. The shopkeepers Sanjeev, Rohit and Deepak spent ₹ 250, ₹ 270 and ₹ 200 on these carry bags respectively.

Based on the above information, solve the following questions:

Q 1. Find the cost of one polythene bag, one handmade bag and one newspaper envelop.

Q 2. From the matrix equation $AB = AC$, it can be concluded that $B = C$, show that A is non-singular.

Solutions

1. Let the cost of a polythene bag = ₹ x
the cost of a handmade bag = ₹ y
and the cost of a newspaper bag = ₹ z
According to the questions,

$$20x + 30y + 40z = 250$$
$$30x + 40y + 20z = 270$$
$$40x + 20y + 30z = 200$$

This system can be written as $AX = B$, where

$$A = \begin{bmatrix} 20 & 30 & 40 \\ 30 & 40 & 20 \\ 40 & 20 & 30 \end{bmatrix}, X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \text{ and } B = \begin{bmatrix} 250 \\ 270 \\ 200 \end{bmatrix}$$

$$\text{Now, } |A| = \begin{vmatrix} 20 & 30 & 40 \\ 30 & 40 & 20 \\ 40 & 20 & 30 \end{vmatrix}$$

$= 20(1200 - 400) - 30(900 - 800) + 40(600 - 1600)$
$= 20(800) - 30(100) + 40(-1000)$
$= 16000 - 3000 - 40000 = -27000 \neq 0$

So, A^{-1} exists and system has a solution given by $X = A^{-1}B$.

$$\text{Now, adj } A = \begin{bmatrix} 800 & -100 & -1000 \\ -100 & -1000 & 800 \\ -1000 & 800 & -100 \end{bmatrix}^T$$

$$= \begin{bmatrix} 800 & -100 & -1000 \\ -100 & -1000 & 800 \\ -1000 & 800 & -100 \end{bmatrix}$$

$$\therefore \quad A^{-1} = \frac{1}{|A|}(\text{adj } A)$$

$$= \frac{1}{-27000}\begin{bmatrix} 800 & -100 & -1000 \\ -100 & -1000 & 800 \\ -1000 & 800 & -100 \end{bmatrix}$$

Now, $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{27000}\begin{bmatrix} -800 & 100 & 1000 \\ 100 & 1000 & -800 \\ 1000 & -800 & 100 \end{bmatrix}\begin{bmatrix} 250 \\ 270 \\ 200 \end{bmatrix}$

$$= \frac{1}{27000}\begin{bmatrix} 27000 \\ 135000 \\ 54000 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 5 \\ 2 \end{bmatrix}$$

$\Rightarrow \quad x = 1, y = 5, z = 2$

Hence, cost of a polythene bag, a handmade bag and a newspaper envelope is ₹ 1, ₹ 5 and ₹ 2 respectively.

2. Given matrix equation is $AB = AC$.

Pre-multiplying by A^{-1} on both sides, we get

$$A^{-1}AB = A^{-1}AC$$

$\Rightarrow \quad (A^{-1}A)B = (A^{-1}A)C$

$\Rightarrow \quad IB = IC \quad [\because AA^{-1} = A^{-1}A = I]$

$\Rightarrow \quad B = C$

Since A^{-1} exists only if A is non-singular.

$\therefore$ For $B = C$, A should be non-singular. **Hence proved.**

Case Study 7

Area of a triangle whose vertices are $(x_1, y_1), (x_2, y_2)$ and (x_3, y_3) is given by the determinant

$$\Delta = \frac{1}{2}\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix}$$

Since, area is a positive quantity, so we always take the absolute value of the determinant Δ. Also, the area of the triangle formed by three collinear points is zero.

Based on the above information, solve the following questions:

Q 1. Find the area of the triangle whose vertices are $(-2, 6)$, $(3, -6)$ and $(1, 5)$.

Q 2. If the points $(2, -3)$, $(k, -1)$ and $(0, 4)$ are collinear, then find the value of $4k$.

Or

If the area of a triangle ABC, with vertices $A(1, 3)$, $B(0, 0)$ and $C(k, 0)$ is 3 sq. units, then find the value of k.

Q 3. Using determinants, find the equation of the line joining the points $A(1, 2)$ and $B(3, 6)$.

Solutions

1. Let Δ be the area of the triangle then

$$\Delta = \frac{1}{2}\left\|\begin{matrix} -2 & 6 & 1 \\ 3 & -6 & 1 \\ 1 & 5 & 1 \end{matrix}\right\|$$

$$= \frac{1}{2}|-2(-6-5) - 6(3-1) + 1(15+6)|$$

[expanding along R_1]

$\Rightarrow \quad \Delta = \frac{1}{2}|43 - 12| = 15.5$ sq. units

2. The given points are collinear.

$$\frac{1}{2}\begin{vmatrix} 2 & -3 & 1 \\ k & -1 & 1 \\ 0 & 4 & 1 \end{vmatrix} = 0$$

Expanding along R_1, we get

$$2(-1-4) + 3(k) + 1(4k) = 0$$

$\Rightarrow \quad 7k - 10 = 0 \Rightarrow k = \frac{10}{7} \Rightarrow 4k = \frac{40}{7}$

Or

Area of $\Delta ABC = 3$ sq. units (given)

$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 3 \Rightarrow \begin{vmatrix} 1 & 3 & 1 \\ 0 & 0 & 1 \\ k & 0 & 1 \end{vmatrix} = \pm 6$

Expanding along R_1, we get

$$1(0-0) - 3(0-k) + 1(0-0) = \pm 6$$

$\Rightarrow \quad 3k = \pm 6 \Rightarrow k = \pm 2$

3. Let $Q(x, y)$ be any point on the line joining $A(1, 2)$ and $B(3, 6)$. Then, area of $\Delta ABQ = 0$

$\Rightarrow \quad \frac{1}{2}\begin{vmatrix} 1 & 2 & 1 \\ 3 & 6 & 1 \\ x & y & 1 \end{vmatrix} = 0$

Expanding along R_1, we get

$$1(6-y) - 2(3-x) + 1(3y - 6x) = 0$$

$\Rightarrow \quad 6 - y - 6 + 2x + 3y - 6x = 0$

$\Rightarrow \quad -4x = -2y \Rightarrow 2x = y$

Case Study 8

Let $A = \begin{bmatrix} 1 & 0 \\ 2 & 1 \end{bmatrix}$ and U_1, U_2 are first and second columns respectively of a 2×2 matrix U. Also, let the column matrices U_1 and U_2 satisfying $AU_1 = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$ and $AU_2 = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$.

Based on the above information, solve the following questions:

Q 1. Find the matrix $U_1 + U_2$.

Q 2. Find the value of $|U|$.

Or

Find the minor of element at the position a_{22} in U.

Q 3. If $X = [3 \;\; 2]\; U \begin{bmatrix} 3 \\ 2 \end{bmatrix}$, then the value of $[X]$.

Solutions

1. We have, $A=\begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}$

Let $U_1=\begin{bmatrix}a\\ b\end{bmatrix}$, then $AU_1=\begin{bmatrix}1\\ 0\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}a\\ b\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix} \Rightarrow \begin{bmatrix}a\\ 2a+b\end{bmatrix}=\begin{bmatrix}1\\ 0\end{bmatrix}$

$\Rightarrow a=1$ and $2a+b=0 \Rightarrow a=1$ and $b=-2$.

Let $U_2=\begin{bmatrix}c\\ d\end{bmatrix}$, then $AU_2=\begin{bmatrix}2\\ 3\end{bmatrix}$

$\Rightarrow \begin{bmatrix}1 & 0\\ 2 & 1\end{bmatrix}\begin{bmatrix}c\\ d\end{bmatrix}=\begin{bmatrix}2\\ 3\end{bmatrix} \Rightarrow \begin{bmatrix}c\\ 2c+d\end{bmatrix}=\begin{bmatrix}2\\ 3\end{bmatrix}$

$\Rightarrow c=2$ and $2c+d=3$

$\Rightarrow c=2$ and $d=3-4=-1$

Thus, $U_1+U_2=\begin{bmatrix}1\\ -2\end{bmatrix}+\begin{bmatrix}2\\ -1\end{bmatrix}=\begin{bmatrix}3\\ -3\end{bmatrix}$

2. Clearly, $U=\begin{bmatrix}1 & 2\\ -2 & -1\end{bmatrix}$

$\therefore \quad |U|=\begin{vmatrix}1 & 2\\ -2 & -1\end{vmatrix}=-1+4=3$

Or

a_{22} in U is -1 and its minor is 1.

3. We have, $X=[3 \;\; 2]\begin{bmatrix}1 & 2\\ -2 & -1\end{bmatrix}\begin{bmatrix}3\\ 2\end{bmatrix}=[3 \;\; 2]\begin{bmatrix}7\\ -8\end{bmatrix}$

$=[21-16]=[5]$

$\therefore \quad [X]=5$

Very Short Answer Type Questions

Q 1. If $A=\begin{bmatrix}1 & 0 & 1\\ 0 & 1 & 2\\ 0 & 0 & 4\end{bmatrix}$. Show that $|3A|=27|A|$. *(NCERT EXERCISE)*

Q 2. If A and B are square matrices of the same order 3, such that $|A|=2$ and $AB=2I$, write the value of $|B|$. *(CBSE 2019)*

Q 3. Find the cofactors of all the elements $\begin{bmatrix}1 & -2\\ 4 & 3\end{bmatrix}$. *(CBSE 2020)*

Q 4. If for any 2×2 square matrix A, $A\,(\text{adj}\,A)=\begin{bmatrix}8 & 0\\ 0 & 8\end{bmatrix}$, then write the value of $|A|$. *(CBSE 2017)*

Q 5. If A is a 3×3 invertible matrix, then what will be the value of k if $\det(A)^{-1}=(\det A)^k$. *(CBSE 2017)*

Q 6. If $|A|=3$ and $A^{-1}=\begin{bmatrix}3 & -1\\ -\frac{5}{3} & \frac{2}{3}\end{bmatrix}$, then write the adj A. *(CBSE 2017)*

Short Answer Type-I Questions

Q 1. If A is a skew-symmetric matrix of order 3, then prove that $\det A=0$. *(CBSE 2017, 20)*

Q 2. If A and B are square matrices of order 3 such that $|A|=-1, |B|=3$, then find the value of $|2AB|$. *(CBSE 2017)*

Q 3. Show that the points $(a, b+c)$, $(b, c+a)$ and $(c, a+b)$ are collinear, using determinants. *(NCERT EXERCISE)*

Q 4. Prove that the inverse of a non-singular square matrix is unique.

Q 5. Prove that $(A^{-1})'=(A')^{-1}$, where A is a non-singular matrix.

Short Answer Type-II Questions

Q 1. Let $A=\begin{bmatrix}1 & \sin\theta & 1\\ -\sin\theta & 1 & \sin\theta\\ -1 & -\sin\theta & 1\end{bmatrix}$, where $0\le\theta\le 2\pi$ then find the determinant of A.

Q 2. Evaluate $\begin{vmatrix}l_1 & m_1 & n_1\\ l_2 & m_2 & n_2\\ l_3 & m_3 & n_3\end{vmatrix}$, where $l_1^2+m_1^2+n_1^2=1$ and $l_1l_2+m_1m_2+n_1n_2=0$ etc.

Q 3. Show that the determinant $\begin{vmatrix}x & \sin\theta & \cos\theta\\ -\sin\theta & -x & 1\\ \cos\theta & 1 & x\end{vmatrix}$ is independent of θ. *(CBSE 2023)*

Q 4. Using determinants, find the area of ΔPQR with vertices $P(3, 1)$, $Q(9, 3)$ and $R(5, 7)$. Also, find the equation of line PQ using determinants. *(CBSE 2023)*

Q 5. Find the inverse of the matrix $A=\begin{bmatrix}1 & 2\\ 2 & -1\end{bmatrix}$.

Q 6. If A and B are two invertible square matrices of order n, then prove that $(AB)^{-1}=B^{-1}A^{-1}$.

Q 7. If $A=\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$ and $B=\begin{bmatrix}a & c\\ b & d\end{bmatrix}$, then find B^{-1} and show that $(A\cdot B)^{-1}=B^{-1}\cdot A^{-1}$.

Long Answer Type Questions

Q 1. Find the cofactors of matrix $A=\begin{bmatrix}-1 & -2 & 3\\ -2 & 2 & 1\\ 4 & -5 & 2\end{bmatrix}$ and prove that $(\text{adj}\,A)\,A=A\,(\text{adj}\,A)$.

Q 2. If $A=\begin{bmatrix}1&3&3\\1&4&3\\1&3&4\end{bmatrix}$, then verify that $A\cdot \text{adj}\,A=|A|I$.

Also find A^{-1}.

Q 3. Find the inverse of the matrix $\begin{bmatrix}1&3&-2\\-3&0&-5\\2&5&0\end{bmatrix}$.

Q 4. If $A^{-1}=\begin{bmatrix}3&-1&1\\-15&6&-5\\5&-2&2\end{bmatrix}$ and $B=\begin{bmatrix}1&2&-2\\-1&3&0\\0&-2&1\end{bmatrix}$, then find $(AB)^{-1}$. *(NCERT EXERCISE)*

Q 5. Solve the following system of equations by matrix method:

$x+y+z=6, y+3z=11, x+z=2y$ *(NCERT EXERCISE)*

Q 6. If $A=\begin{bmatrix}1&-1&0\\2&3&4\\0&1&2\end{bmatrix}$ and $B=\begin{bmatrix}2&2&-4\\-4&2&-4\\2&-1&5\end{bmatrix}$ are two square matrices, find AB and hence solve the system of linear equations

$x-y=3, 2x+3y+4z=17$ and $y+2z=7$.

(NCERT EXEMPLAR; CBSE 2017)

Q 7. Use product $\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}\begin{bmatrix}-2&0&1\\9&2&-3\\6&1&-2\end{bmatrix}$ to solve the system of equations $x+3z=9, -x+2y-2z=4, 2x-3y+4z=-3$. *(CBSE 2017)*

Q 8. If $A=\begin{bmatrix}2&-3&5\\3&2&-4\\1&1&-2\end{bmatrix}$, find A^{-1}.

Using A^{-1}, solve the following system of equations:

$2x-3y+5z=11, 3x+2y-4z=-5, x+y-2z=-3$.

(CBSE 2020, 19, 18, 17; CBSE SQP 2022-23)

Q 9. Find the inverse of the matrix $A=\begin{bmatrix}1&-1&2\\0&2&-3\\3&-2&4\end{bmatrix}$.

Using the inverse, A^{-1}, solve the system of linear equations

$x-y+2z=1;\ 2y-3z=1;\ 3x-2y+4z=3.$ *(CBSE 2017)*

Q 10. Using the matrix method, solve the following system of linear equations:

$\frac{2}{x}+\frac{3}{y}+\frac{10}{z}=4,\ \frac{4}{x}-\frac{6}{y}+\frac{5}{z}=1,\ \frac{6}{x}+\frac{9}{y}-\frac{20}{z}=2.$

(CBSE SQP 2023-24)

Solutions

Very Short Answer Type Questions

1. $3A=\begin{bmatrix}3&0&3\\0&3&6\\0&0&12\end{bmatrix}$

$\Rightarrow\quad |3A|=3(36-0)=108$

and $\quad 27|A|=27(4-0)=108$

$\therefore\quad |3A|=27|A|$ **Hence proved.**

2. Given, A and B are square matrices of the same order 3, such that $|A|=2$ and $AB=2I$

Taking determinant on both sides, we get

$|AB|=|2I|$

TR!CK

$|AB|=|A|\cdot|B|$ and $|kA|=k^n|A|$, where n is the order of matrix.

$\Rightarrow\quad |A|\cdot|B|=2^3\cdot|I|$

$\Rightarrow\quad 2\cdot|B|=8\cdot 1 \quad [\because |I|=1]$

$\therefore\quad |B|=4$

COMMON ERROR

Generally students solve $|aA|$ as:

$|aA|=a|A|$

But this is wrong and leads to incorrect solution.

3. Let $A=\begin{bmatrix}1&-2\\4&3\end{bmatrix}$

TR!CKS

- *Minor of an element a_{ij} of the determinant is the determinant obtained by deleting the ith row and jth column in which element a_{ij} lies. Minor of an element a_{ij} is denoted by M_{ij}.*
- *Cofactor of an element a_{ij} of a determinant is denoted by A_{ij} or C_{ij} and is defined as $A_{ij}=(-1)^{i+j}M_{ij}$.*

Cofactors of matrix A are given below:

$A_{11}=(-1)^{1+1}(3)=(-1)^2\times 3=1\times 3=3$

$A_{12}=(-1)^{1+2}(4)=(-1)^3\times 4=-1\times 4=-4$

$A_{21}=(-1)^{2+1}(-2)=(-1)^3\times(-2)=-1\times(-2)=2$

$A_{22}=(-1)^{2+2}(1)=(-1)^4\times 1=1\times 1=1$

So, cofactors of given matrix are 3, −4, 2 and 1.

4. Given that, $A(\text{adj}\,A)=\begin{bmatrix}8&0\\0&8\end{bmatrix}$

By using property, $A(\text{adj}\,A)=|A|I_n$

TiP

Here, A is 2×2 square matrix, thus the order of identity matrix is also 2×2.

$$\therefore \quad |A|I_2 = \begin{bmatrix} 8 & 0 \\ 0 & 8 \end{bmatrix} \Rightarrow |A|I_2 = 8\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} = 8I_2$$

$\therefore \quad |A| = 8$

5.

TR!CK

$|AB| = |A|\,|B|$

or det (AB) = det (A) · det (B)

Given, A is a 3×3 invertible matrix.

$\therefore \quad AA^{-1} = I \Rightarrow \det(A \cdot A^{-1}) = \det(I)$

$\Rightarrow \quad \det(A) \cdot \det(A^{-1}) = 1 \quad [\because \det(I) = 1]$

$\Rightarrow \quad \det(A^{-1}) = \frac{1}{\det(A)}$

$\Rightarrow \quad \det(A^{-1}) = (\det A)^{-1}$

On comparing above eq. with $\det(A^{-1}) = (\det A)^k$, we get

$k = -1$

6. Given, $|A| = 3$ and $A^{-1} = \begin{bmatrix} 3 & -1 \\ -\frac{5}{3} & \frac{2}{3} \end{bmatrix}$

We know that,

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$\Rightarrow \quad \text{adj } A = |A|\, A^{-1} = 3 \cdot \begin{bmatrix} 3 & -1 \\ -\frac{5}{3} & \frac{2}{3} \end{bmatrix} = \begin{bmatrix} 9 & -3 \\ -5 & 2 \end{bmatrix}$$

Short Answer Type-I Questions

1. Let $A = \begin{bmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{bmatrix}$ be a skew-symmetric matrix of order 3.

TiP

If A be a skew-symmetric matrix, then $A = -A^T$

$$\therefore \quad |A| \text{ or } \det A = \begin{vmatrix} 0 & a & b \\ -a & 0 & c \\ -b & -c & 0 \end{vmatrix} \quad [\text{expand along } R_1]$$

$= 0\,(0 + c^2) - a\,(0 + bc) + b\,(ac + 0)$

$= 0 - abc + abc = 0$ **Hence proved.**

2.

TR!CK

$|aA| = a^k\,|A|$ where, k is the order of matrix A.

Given that, A and B are square matrices of order 3 such that $|A| = -1$ and $|B| = 3$.

$\therefore \quad |2AB| = 2^3\,|AB| \quad [\because |AB| = |A|\,|B|]$

$= 8\,|A|\,|B| = 8 \times -1 \times 3 = -24$

COMMON ERR(!)R

Generally students make mistakes in simplifying $|aAB|$.
They solve $|aAB|$ as: $|aAB| = a\,|AB| = a\,|A|\,|B|$
But this is wrong.

3.

TiP

The area of triangle formed by three collinear points is zero.

Area of triangle formed by the given points

$$= \frac{1}{2}\begin{vmatrix} a & b+c & 1 \\ b & c+a & 1 \\ c & a+b & 1 \end{vmatrix}$$

Expanding along R_1

$= \frac{1}{2}[a(c+a-a-b) - (b+c)(b-c) + 1(ab + b^2 - c^2 - ac)]$

$= \frac{1}{2}[ac - ab - (b^2 - c^2) + ab + b^2 - c^2 - ac]$

$= \frac{1}{2}[0] = 0$

Therefore, the given points are collinear.

Hence proved.

4. Let A be a non-singular matrix of order n.

Let B and C be two inverse of matrix A.

Then, $AB = BA = I$ ...(1)

and $AC = CA = I_n$ ...(2)

From eq. (1), $AB = I_n$

Multiply the both sides by C from left

$C\,(AB) = CI_n \Rightarrow C\,(AB) = C \quad [\because CI_n = C]$

$\Rightarrow \quad (CA)\,B = C$

[from associative law of multiplication of matrices]

$\Rightarrow \quad I_nB = C$ [from eq. (2)]

$\Rightarrow \quad B = C \quad [\because I_nB = B]$

Hence, the inverse of A is unique. **Hence proved.**

5. $\because A$ is a non-singular matrix so $|A| \neq 0$.

We know that, $|A| = |A'|$ but $|A| \neq 0$

So $|A'| \neq 0$ i.e., A' is also a non-singular matrix.

We know that $AA^{-1} = A^{-1}A = I$

Taking transpose on both sides, we get

$(A^{-1})'\,A' = A'\,(A^{-1})' = (I)' = I$

Therefore, $(A^{-1})'$ is the inverse of matrix A'.

$\Rightarrow \quad (A')^{-1} = (A^{-1})'$ **Hence proved.**

Short Answer Type-II Questions

1. Given, $A = \begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$

$$\therefore \quad |A| = \begin{vmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{vmatrix}$$

[expanding along R_1]

$= 1(1 + \sin^2\theta) - \sin\theta(-\sin\theta + \sin\theta) + 1(\sin^2\theta + 1)$

$= 1 + \sin^2\theta + 0 + \sin^2\theta + 1$

$= 2 + 2\sin^2\theta = 2\,(1 + \sin^2\theta)$

When $\theta = 0, \pi, 2\pi$, then $\sin\theta = 0$

$\Rightarrow \quad |A| = 2 + 2\sin^2 0 = 2$

When $\theta = \frac{\pi}{2}, \frac{3\pi}{2}$ then $\sin^2\theta = 1$

$|A| = 2\,(1 + 1) = 2 \times 2 = 4$

Here, we see that on multiple angle of $\frac{\pi}{2}$, we get determinant 2 and 4, but in other angles we get the determinant lies between 2 and 4.

Hence $\quad \det(A) \in [2, 4]$

2. Now, $\begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}^2 = \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} \times \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix}$

$$= \begin{vmatrix} l_1^2+m_1^2+n_1^2 & l_1l_2+m_1m_2+n_1n_2 & l_1l_3+m_1m_3+n_1n_3 \\ l_1l_2+m_1m_2+n_1n_2 & l_2^2+m_2^2+n_2^2 & l_2l_3+m_2m_3+n_2n_3 \\ l_1l_3+m_1m_3+n_1n_3 & l_2l_3+m_2m_3+n_2n_3 & l_3^2+m_3^2+n_3^2 \end{vmatrix}$$

$= \begin{vmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{vmatrix}$ $\quad [\because l_i^2+m_i^2+n_i^2 = 1$ for all $i = 1, 2, 3$; $l_il_j+m_im_j+n_in_j = 0$ for all $i = 1, 2, 3$ and $j = 1, 2, 3$ where $i \neq j]$

$= 1(1-0)-0+0 = 1$ $\quad$ [expand along R_1]

$\therefore \quad \begin{vmatrix} l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \\ l_3 & m_3 & n_3 \end{vmatrix} = \pm 1$

3. We have $\Delta = \begin{vmatrix} x & \sin\theta & \cos\theta \\ -\sin\theta & -x & 1 \\ \cos\theta & 1 & x \end{vmatrix}$

Expanding along R_1,

$\Delta = x(-x^2-1) - \sin\theta(-x\sin\theta - \cos\theta) + \cos\theta(-\sin\theta + x\cos\theta)$

$= -x^3 - x + x\sin^2\theta + \sin\theta\cos\theta - \sin\theta\cos\theta + x\cos^2\theta$

$= -x^3 - x + x(\sin^2\theta + \cos^2\theta)$

$= -x^3 - x + x(1)$ $\quad [\because \sin^2\theta + \cos^2\theta = 1]$

$= -x^3$

Hence, it shows that given determinant is independent of θ.

4. Area of triangle with vertices $P(3, 1)$, $Q(9, 3)$ and $R(5, 7)$

$= \frac{1}{2}\begin{vmatrix} 3 & 1 & 1 \\ 9 & 3 & 1 \\ 5 & 7 & 1 \end{vmatrix}$

$= \frac{1}{2}[3(3-7) - 1(9-5) + 1(63-15)]$

$= \frac{1}{2}[-12-4+48] = \frac{32}{2} = 16$ sq. units.

TR!CK

The equation of line passing through two points $A(x_1, y_1)$ and $B(x_2, y_2)$ is

$$\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$$

Now, equation of line PQ is

$$\begin{vmatrix} x & y & 1 \\ 3 & 1 & 1 \\ 9 & 3 & 1 \end{vmatrix} = 0$$

Expanding along R_1,

$\Rightarrow \quad x(1-3) - y(3-9) + 1(9-9) = 0$

$\Rightarrow \quad x(-2) - y(-6) + 0 = 0$

$\Rightarrow \quad -2x + 6y = 0$

5. Given $A = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$

Cofactors of the elements of A is given below:

$A_{11} = (-1)^2(-1) = -1, \quad A_{12} = (-1)^3(2) = -2$

$A_{21} = (-1)^3(2) = -2$ and $A_{22} = (-1)^4(1) = 1$

$\therefore$ Matrix formed by the cofactors of the elements of A:

$$B = \begin{bmatrix} A_{11} & A_{12} \\ A_{21} & A_{22} \end{bmatrix} = \begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$$

$\Rightarrow \quad \text{adj } A = B' = \begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}' = \begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix}$

Now, $\quad |A| = \begin{vmatrix} 1 & 2 \\ 2 & -1 \end{vmatrix} = -1-4 = -5 \neq 0$

$\therefore \quad A^{-1} = \frac{\text{adj } A}{|A|} = -\frac{1}{5}\begin{bmatrix} -1 & -2 \\ -2 & 1 \end{bmatrix} = \begin{bmatrix} 1/5 & 2/5 \\ 2/5 & -1/5 \end{bmatrix}$

6. A and B are invertible matrices

$\therefore \quad |A| \neq 0$ and $|B| \neq 0$

$\Rightarrow \quad |A||B| \neq 0$

$\Rightarrow \quad |AB| \neq 0$ $\quad [\because |AB| = |A||B|]$

So, (AB) is an invertible matrix.

Now, $(AB)(B^{-1}A^{-1}) = A(BB^{-1})A^{-1}$

[from associative law of multiplication of matrices]

$= (AI)A^{-1}$ $\quad [\because BB^{-1} = I]$

$= AA^{-1}$ $\quad [\because AI = A]$

$= I$ $\quad [\because AA^{-1} = I]$

and $(B^{-1}A^{-1})(AB) = B^{-1}(A^{-1}A)B = B^{-1}(IB)$

$= B^{-1}B = I$

$\therefore \quad (AB)(B^{-1}A^{-1}) = (B^{-1}A^{-1})(AB) = I$

Hence, $\quad (AB)^{-1} = B^{-1}A^{-1}$ $\quad$ **Hence proved.**

7. $\because A$ is a unit matrix, so $A^{-1} = I$

and for matrix B, $|B| = \begin{vmatrix} a & c \\ b & d \end{vmatrix} = ad - bc$

and the cofactor of different elements are:

$B_{11} = d, B_{12} = -b, B_{21} = -c, B_{22} = a$

$\therefore$ Matrix formed from cofactors

$$C = \begin{bmatrix} d & -b \\ -c & a \end{bmatrix}$$

$\therefore \quad \text{adj } B = C' = \begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$

$\therefore \quad B^{-1} = \frac{\text{adj } B}{|B|} = \frac{1}{ad-bc}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$

Now $\quad AB = B$ $\quad [\because A = I]$

$\Rightarrow \quad (AB)^{-1} = B^{-1} = \frac{1}{ad-bc}\begin{bmatrix} d & -c \\ -b & a \end{bmatrix}$ $\quad$ ...(1)

and $B^{-1}A^{-1} = B^{-1}I = B^{-1}$ ...(2)

From eqs. (1) and (2),

$\therefore$ $(AB)^{-1} = B^{-1}A^{-1}$ **Hence proved.**

Long Answer Type Questions

1. Cofactors of the elements of A

$$A_{11} = (-1)^2 \begin{vmatrix} 2 & 1 \\ -5 & 2 \end{vmatrix} = (4+5) = 9$$

$$A_{12} = (-1)^3 \begin{vmatrix} -2 & 1 \\ 4 & 2 \end{vmatrix} = -(-4-4) = 8$$

$$A_{13} = (-1)^4 \begin{vmatrix} -2 & 2 \\ 4 & -5 \end{vmatrix} = (10-8) = 2$$

$$A_{21} = (-1)^3 \begin{vmatrix} -2 & 3 \\ -5 & 2 \end{vmatrix} = -(-4+15) = -11$$

$$A_{22} = (-1)^4 \begin{vmatrix} -1 & 3 \\ 4 & 2 \end{vmatrix} = (-2-12) = -14$$

$$A_{23} = (-1)^5 \begin{vmatrix} -1 & -2 \\ 4 & -5 \end{vmatrix} = -(5+8) = -13$$

$$A_{31} = (-1)^4 \begin{vmatrix} -2 & 3 \\ 2 & 1 \end{vmatrix} = (-2-6) = -8$$

$$A_{32} = (-1)^5 \begin{vmatrix} -1 & 3 \\ -2 & 1 \end{vmatrix} = -(-1+6) = -5$$

$$A_{33} = (-1)^6 \begin{vmatrix} -1 & -2 \\ -2 & 2 \end{vmatrix} = (-2-4) = -6$$

$$C = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 9 & 8 & 2 \\ -11 & -14 & -13 \\ -8 & -5 & -6 \end{bmatrix}$$

$$\therefore \quad \text{adj } A = C^T = \begin{bmatrix} 9 & 8 & 2 \\ -11 & -14 & -13 \\ -8 & -5 & -6 \end{bmatrix}^T = \begin{bmatrix} 9 & -11 & -8 \\ 8 & -14 & -5 \\ 2 & -13 & -6 \end{bmatrix}$$

$$\text{LHS} = (\text{adj } A)\, A = \begin{bmatrix} 9 & -11 & -8 \\ 8 & -14 & -5 \\ 2 & -13 & -6 \end{bmatrix} \begin{bmatrix} -1 & -2 & 3 \\ -2 & 2 & 1 \\ 4 & -5 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} -9+22-32 & -18-22+40 & 27-11-16 \\ -8+28-20 & -16-28+25 & 24-14-10 \\ -2+26-24 & -4-26+30 & 6-13-12 \end{bmatrix}$$

$$= \begin{bmatrix} -19 & 0 & 0 \\ 0 & -19 & 0 \\ 0 & 0 & -19 \end{bmatrix} = -19 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = -19I$$

$$\text{RHS} = A\,(\text{adj } A) = \begin{bmatrix} -1 & -2 & 3 \\ -2 & 2 & 1 \\ 4 & -5 & 2 \end{bmatrix} \begin{bmatrix} 9 & -11 & -8 \\ 8 & -14 & -5 \\ 2 & -13 & -6 \end{bmatrix}$$

$$= \begin{bmatrix} -9-16+6 & 11+28-39 & 8+10-18 \\ -18+16+2 & 22-28-13 & 16-10-6 \\ 36-40+4 & -44+70-26 & -32+25-12 \end{bmatrix}$$

$$= \begin{bmatrix} -19 & 0 & 0 \\ 0 & -19 & 0 \\ 0 & 0 & -19 \end{bmatrix} = -19 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = -19I$$

$\therefore$ LHS = RHS **Hence proved.**

2. $$|A| = \begin{vmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{vmatrix} = 1(16-9) - 3(4-3) + 3(3-4)$$

$$= 7-3-3 = 7-6 = 1 \neq 0$$

Cofactors of the elements of matrix A are:

$$A_{11} = \begin{vmatrix} 4 & 3 \\ 3 & 4 \end{vmatrix} = 7,\ A_{12} = -\begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = -1,\ A_{13} = \begin{vmatrix} 1 & 4 \\ 1 & 3 \end{vmatrix} = -1$$

$$A_{21} = -\begin{vmatrix} 3 & 3 \\ 3 & 4 \end{vmatrix} = -3,\ A_{22} = \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 1,\ A_{23} = -\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} = 0$$

$$A_{31} = \begin{vmatrix} 3 & 3 \\ 4 & 3 \end{vmatrix} = -3,\ A_{32} = -\begin{vmatrix} 1 & 3 \\ 1 & 3 \end{vmatrix} = 0,\ A_{33} = \begin{vmatrix} 1 & 3 \\ 1 & 4 \end{vmatrix} = 1$$

$\therefore$ Matrix formed from cofactor is:

$$B = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 7 & -1 & -1 \\ -3 & 1 & 0 \\ -3 & 0 & 1 \end{bmatrix}$$

$$\Rightarrow \text{adj } A = \text{transpose of } B = B' = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$\text{Now } A\,(\text{adj } A) = \begin{bmatrix} 1 & 3 & 3 \\ 1 & 4 & 3 \\ 1 & 3 & 4 \end{bmatrix} \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 7-3-3 & -3+3+0 & -3+0+3 \\ 7-4-3 & -3+4+0 & -3+0+3 \\ 7-3-4 & -3+3+0 & -3+0+4 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = (1) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = |A| \cdot I$$

$$\text{Hence, } A^{-1} = \frac{1}{|A|} \cdot \text{adj } A = \begin{bmatrix} 7 & -3 & -3 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix} \qquad [\because |A| = 1]$$

3. Let the given matrix be A, then

$$|A| = \begin{vmatrix} 1 & 3 & -2 \\ -3 & 0 & -5 \\ 2 & 5 & 0 \end{vmatrix} = 1(0+25) - 3(0+10) + (-2)(-15-0)$$

$$= 25 - 30 + 30 = 25 \neq 0$$

Cofactors of the elements of the matrix A are:

$$A_{11} = \begin{vmatrix} 0 & -5 \\ 5 & 0 \end{vmatrix} = 0+25 = 25$$

$$A_{12} = -\begin{vmatrix} -3 & -5 \\ 2 & 0 \end{vmatrix} = -(0+10) = -10$$

$$A_{13} = \begin{vmatrix} -3 & 0 \\ 2 & 5 \end{vmatrix} = -15-0 = -15$$

$$A_{21} = -\begin{vmatrix} 3 & -2 \\ 5 & 0 \end{vmatrix} = -(0+10) = -10$$

$$A_{22} = \begin{vmatrix} 1 & -2 \\ 2 & 0 \end{vmatrix} = 0+4 = 4$$

$$A_{23} = -\begin{vmatrix} 1 & 3 \\ 2 & 5 \end{vmatrix} = -(5-6) = 1$$

$$A_{31} = \begin{vmatrix} 3 & -2 \\ 0 & -5 \end{vmatrix} = -15 - 0 = -15$$

$$A_{32} = -\begin{vmatrix} 1 & -2 \\ -3 & -5 \end{vmatrix} = -(-5-6) = 11$$

$$A_{33} = \begin{vmatrix} 1 & 3 \\ -3 & 0 \end{vmatrix} = 0 + 9 = 9$$

∴ Matrix formed by cofactors:

$$B = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix} = \begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 1 \\ -15 & 11 & 9 \end{bmatrix}$$

$$\Rightarrow \text{adj } A = B^T = \begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 1 \\ -15 & 11 & 9 \end{bmatrix}^T = \begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$$

Hence, $A^{-1} = \frac{1}{|A|} \text{adj } A = \frac{1}{25}\begin{bmatrix} 25 & -10 & -15 \\ -10 & 4 & 11 \\ -15 & 1 & 9 \end{bmatrix}$

4. Given, $B = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix} \Rightarrow |B| = \begin{vmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{vmatrix}$

$|B| = 1(3-0) - 2(-1-0) - 2(2-0) = 3 + 2 - 4 = 1$

Cofactors of the elements of the matrix B are:

$$B_{11} = (-1)^2\begin{vmatrix} 3 & 0 \\ -2 & 1 \end{vmatrix} = 3, B_{12} = (-1)^3\begin{vmatrix} -1 & 0 \\ 0 & 1 \end{vmatrix} = 1$$

$$B_{13} = (-1)^4\begin{vmatrix} -1 & 3 \\ 0 & -2 \end{vmatrix} = 2, B_{21} = (-1)^3\begin{vmatrix} 2 & -2 \\ -2 & 1 \end{vmatrix} = 2$$

$$B_{22} = (-1)^4\begin{vmatrix} 1 & -2 \\ 0 & 1 \end{vmatrix} = 1, B_{23} = (-1)^5\begin{vmatrix} 1 & 2 \\ 0 & -2 \end{vmatrix} = 2$$

$$B_{31} = (-1)^4\begin{vmatrix} 2 & -2 \\ 3 & 0 \end{vmatrix} = 6, B_{32} = (-1)^5\begin{vmatrix} 1 & -2 \\ -1 & 0 \end{vmatrix} = 2$$

$$B_{33} = (-1)^6\begin{vmatrix} 1 & 2 \\ -1 & 3 \end{vmatrix} = 5$$

∴ Matrix formed by cofactor of $B = \begin{bmatrix} 3 & 1 & 2 \\ 2 & 1 & 2 \\ 6 & 2 & 5 \end{bmatrix} = C(\text{say})$

∴ $\text{adj } B = C' = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Now, $B^{-1} = \frac{\text{adj } B}{|B|} = \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}$

Given, $A^{-1} = \begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$

Therefore, $(AB)^{-1} = B^{-1}A^{-1}$

$$= \begin{bmatrix} 3 & 2 & 6 \\ 1 & 1 & 2 \\ 2 & 2 & 5 \end{bmatrix}\begin{bmatrix} 3 & -1 & 1 \\ -15 & 6 & -5 \\ 5 & -2 & 2 \end{bmatrix}$$

$$= \begin{bmatrix} 9-30+30 & -3+12-12 & 3-10+12 \\ 3-15+10 & -1+6-4 & 1-5+4 \\ 6-30+25 & -2+12-10 & 2-10+10 \end{bmatrix}$$

$$= \begin{bmatrix} 9 & -3 & 5 \\ -2 & 1 & 0 \\ 1 & 0 & 2 \end{bmatrix}$$

5. Given system of equations: $x + y + z = 6, y + 3z = 11$

or $0 \cdot x + y + 3z = 11$ and $x + z = 2y$ or $x - 2y + z = 0$

The above system of equations can be written in the following matrix form:

$AX = B$

$$\begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

where, $A = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{bmatrix}, x = \begin{bmatrix} x \\ y \\ z \end{bmatrix}$ and $B = \begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$

Here, $|A| = \begin{vmatrix} 1 & 1 & 1 \\ 0 & 1 & 3 \\ 1 & -2 & 1 \end{vmatrix}$

$= 1(1+6) - 1(0-3) + 1(0-1)$

$= 7 + 3 - 1 = 10 - 1 = 9 \neq 0$

⇒ System of given equation have a unique solution.

Cofactors of the elements of matrix A are:

$$C_{11} = \begin{vmatrix} 1 & 3 \\ -2 & 1 \end{vmatrix} = 1 + 6 = 7, C_{12} = -\begin{vmatrix} 0 & 3 \\ 1 & 1 \end{vmatrix} = -(0-3) = 3$$

$$C_{13} = \begin{vmatrix} 0 & 1 \\ 1 & -2 \end{vmatrix} = 0 - 1 = -1, C_{21} = -\begin{vmatrix} 1 & 1 \\ -2 & 1 \end{vmatrix} = -(1+2) = -3$$

$$C_{22} = \begin{vmatrix} 1 & 1 \\ 1 & 1 \end{vmatrix} = 1 - 1 = 0, C_{23} = -\begin{vmatrix} 1 & 1 \\ 1 & -2 \end{vmatrix} = -(-2-1) = 3$$

$$C_{31} = \begin{vmatrix} 1 & 1 \\ 1 & 3 \end{vmatrix} = 3 - 1 = 2, C_{32} = -\begin{vmatrix} 1 & 1 \\ 0 & 3 \end{vmatrix} = -(3-0) = -3$$

$$C_{33} = \begin{vmatrix} 1 & 1 \\ 0 & 1 \end{vmatrix} = 1 - 0 = 1$$

$$\therefore \quad \text{adj } A = \begin{bmatrix} 7 & 3 & -1 \\ -3 & 0 & 3 \\ 2 & -3 & 1 \end{bmatrix}^T = \begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

$$\Rightarrow \quad A^{-1} = \frac{1}{|A|} \cdot (\text{adj } A) = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}$$

Now, $X = A^{-1} \cdot B$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 7 & -3 & 2 \\ 3 & 0 & -3 \\ -1 & 3 & 1 \end{bmatrix}\begin{bmatrix} 6 \\ 11 \\ 0 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 42-33+0 \\ 18+0-0 \\ -6+33+0 \end{bmatrix} = \frac{1}{9}\begin{bmatrix} 9 \\ 18 \\ 27 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

Hence, $x = 1, y = 2, z = 3$

6. Given, $A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \times \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix}$$

$$= \begin{bmatrix} 2+4+0 & 2-2-0 & -4+4+0 \\ 4-12+8 & 4+6-4 & -8-12+20 \\ 0-4+4 & 0+2-2 & 0-4+10 \end{bmatrix}$$

$$= \begin{bmatrix} 6 & 0 & 0 \\ 0 & 6 & 0 \\ 0 & 0 & 6 \end{bmatrix} = 6 \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = 6 I_3$$

$\Rightarrow \quad AB = 6 I_3$

Now premultiplying by A^{-1} on both sides, we get

$(A^{-1}A) B = A^{-1} (6 I_3)$

$\Rightarrow \quad I_3 B = 6A^{-1} I_3 \qquad [\because AA^{-1} = I = A^{-1}A]$

$\Rightarrow \quad B = 6A^{-1} \qquad [\because AI = A = IA]$

$\Rightarrow \quad A^{-1} = \frac{1}{6} B$

$$\Rightarrow \quad A^{-1} = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \qquad ...(1)$$

Now, the given system of equations can be written in matrix form as below:

$$\begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \Rightarrow AX = \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\left\{ \because A = \begin{bmatrix} 1 & -1 & 0 \\ 2 & 3 & 4 \\ 0 & 1 & 2 \end{bmatrix} \right\}$$

$$\Rightarrow \quad (A^{-1}A) X = A^{-1} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

{pre-multiply by A^{-1} on both sides}

$$\Rightarrow \quad IX = A^{-1} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \qquad \{\because IX = X\}$$

$$\Rightarrow \quad X = A^{-1} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix}$$

$$\Rightarrow \quad X = \frac{1}{6} \begin{bmatrix} 2 & 2 & -4 \\ -4 & 2 & -4 \\ 2 & -1 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ 17 \\ 7 \end{bmatrix} \qquad \text{[from eq. (1)]}$$

$$\Rightarrow \quad X = \frac{1}{6} \begin{bmatrix} 6+34-28 \\ -12+34-28 \\ 6-17+35 \end{bmatrix} = \frac{1}{6} \begin{bmatrix} 12 \\ -6 \\ 24 \end{bmatrix}$$

$$\Rightarrow \quad X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 12/6 \\ -6/6 \\ 24/6 \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 4 \end{bmatrix}$$

On comparing, we get

$x = 2, \ y = -1$ and $z = 4$

7. Let $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

and $B = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$

$$AB = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix} \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

$$= \begin{bmatrix} -2-9+12 & 0-2+2 & 1+3-4 \\ 0+18-18 & 0+4-3 & 0-6+6 \\ -6-18+24 & 0-4+4 & 3+6-8 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I_3$$

Since, $AB = I$

Now, premultiplying by A^{-1} on both sides,

$(A^{-1}A) B = A^{-1} I \qquad [\because A^{-1}A = I = AA^{-1}]$

$\Rightarrow \quad IB = A^{-1} \qquad [\because A^{-1} I = A^{-1} = IA^{-1}]$

$\Rightarrow \quad B = A^{-1} \qquad [\because IB = B = BI] \ ...(1)$

Now, the matrix form of the given system of equations can be represented as:

$$\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$

Here, it is clear that $\begin{bmatrix} 1 & 0 & 3 \\ -1 & 2 & -2 \\ 2 & -3 & 4 \end{bmatrix} = A^T$

$$A^T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$

$$\Rightarrow \quad (A^T)^{-1} A^T \begin{bmatrix} x \\ y \\ z \end{bmatrix} = (A^T)^{-1} \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$

(On premultiplying by $(A^T)^{-1}$ both sides)

$$\Rightarrow \quad I \begin{bmatrix} x \\ y \\ z \end{bmatrix} = (A^{-1})^T \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$

$[\because (A^T)^{-1} = (A^{-1})^T$ and $(A^T)^{-1}(A^T) = I = (A^T)(A^T)^{-1}]$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = B^T \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix} \qquad \text{[from eq. (1)]}$$

$$= \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}^T \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix} = \begin{bmatrix} -2 & 9 & 6 \\ 0 & 2 & 1 \\ 1 & -3 & -2 \end{bmatrix} \begin{bmatrix} 9 \\ 4 \\ -3 \end{bmatrix}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -18+36-18 \\ 0+8-3 \\ 9-12+6 \end{bmatrix} \Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 \\ 5 \\ 3 \end{bmatrix}$$

On comparing, we get

$x = 0, y = 5$ and $z = 3$

8. Given, matrix $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$

So, det A or $|A| = \begin{vmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{vmatrix}$

$$= 2(-4+4) + 3(-6+4) + 5(3-2)$$
$$= 2 \times 0 + 3(-2) + 5(1)$$
$$= 0 - 6 + 5 = -1 \neq 0$$

$\therefore$ Inverse of A exists.

Now, cofactors of matrix A are given below:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} 2 & -4 \\ 1 & -2 \end{vmatrix} = (-1)^2(-4+4) = 0$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 3 & -4 \\ 1 & -2 \end{vmatrix} = (-1)^3(-6+4) = (-1)(-2) = 2$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 3 & 2 \\ 1 & 1 \end{vmatrix} = (-1)^4(3-2) = 1 \times 1 = 1$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} -3 & 5 \\ 1 & -2 \end{vmatrix} = (-1)^3(6-5) = -1 \times 1 = -1$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 5 \\ 1 & -2 \end{vmatrix} = (-1)^4(-4-5) = 1 \times (-9) = -9$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 2 & -3 \\ 1 & 1 \end{vmatrix} = (-1)^5(2+3) = (-1) \times 5 = -5$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} -3 & 5 \\ 2 & -4 \end{vmatrix} = (-1)^4(12-10) = 1 \times 2 = 2$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 5 \\ 3 & -4 \end{vmatrix} = (-1)^5(-8-15) = (-1) \times (-23) = 23$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 2 & -3 \\ 3 & 2 \end{vmatrix} = (-1)^6(4+9) = 1 \times 13 = 13$$

So; adjoint of A *i.e.*,

$$\text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 0 & 2 & 1 \\ -1 & -9 & -5 \\ 2 & 23 & 13 \end{bmatrix}^T = \begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix}$$

$$A^{-1} = \frac{\text{adj } A}{|A|}$$

$$= -\begin{bmatrix} 0 & -1 & 2 \\ 2 & -9 & 23 \\ 1 & -5 & 13 \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix} \quad \ldots(1)$$

Now, the given system of linear equations can be written in matrix form as below:

$$\begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix}$$

$$\Rightarrow A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \qquad \left\{\because A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}\right\}$$

Premultiplying by A^{-1} on both sides;

$$(A^{-1}A)\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \qquad [\because A^{-1}A = I_3 = AA^{-1}]$$

$$\Rightarrow I_3\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0 & 1 & -2 \\ -2 & 9 & -23 \\ -1 & 5 & -13 \end{bmatrix}\begin{bmatrix} 11 \\ -5 \\ -3 \end{bmatrix} \qquad \text{[from eq. (1)]}$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 0-5+6 \\ -22-45+69 \\ -11-25+39 \end{bmatrix} \qquad [\because AI_3 = A = I_3A]$$

$$\Rightarrow \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$$

On comparing, we get

$$x = 1, y = 2 \text{ and } z = 3$$

9. Given, matrix $A = \begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}$

$$\det(A) = \begin{vmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{vmatrix}$$
$$= 1(8-6) + 1(0+9) + 2(0-6)$$
$$= 2 + 9 - 12 = -1 \neq 0$$

So; inverse of A exist.

Now, cofactors of matrix A are.

$$C_{11} = (-1)^{1+1}\begin{vmatrix} 2 & -3 \\ -2 & 4 \end{vmatrix} = (-1)^2(8-6) = 2$$

$$C_{12} = (-1)^{1+2}\begin{vmatrix} 0 & -3 \\ 3 & 4 \end{vmatrix} = (-1)^3(0+9) = -9$$

$$C_{13} = (-1)^{1+3}\begin{vmatrix} 0 & 2 \\ 3 & -2 \end{vmatrix} = (-1)^4(0-6) = -6$$

$$C_{21} = (-1)^{2+1}\begin{vmatrix} -1 & 2 \\ -2 & 4 \end{vmatrix} = (-1)^3(-4+4) = 0$$

$$C_{22} = (-1)^{2+2}\begin{vmatrix} 1 & 2 \\ 3 & 4 \end{vmatrix} = (-1)^4(4-6) = -2$$

$$C_{23} = (-1)^{2+3}\begin{vmatrix} 1 & -1 \\ 3 & -2 \end{vmatrix} = (-1)^5(-2+3) = -1$$

$$C_{31} = (-1)^{3+1}\begin{vmatrix} -1 & 2 \\ 2 & -3 \end{vmatrix} = (-1)^4(3-4) = -1$$

$$C_{32} = (-1)^{3+2}\begin{vmatrix} 1 & 2 \\ 0 & -3 \end{vmatrix} = (-1)^5(-3-0) = 3$$

$$C_{33} = (-1)^{3+3}\begin{vmatrix} 1 & -1 \\ 0 & 2 \end{vmatrix} = (-1)^6(2-0) = 2$$

So, adjoint of A *i.e.*,

$$\text{adj } A = \begin{bmatrix} C_{11} & C_{12} & C_{13} \\ C_{21} & C_{22} & C_{23} \\ C_{31} & C_{32} & C_{33} \end{bmatrix}^T$$

$$= \begin{bmatrix} 2 & -9 & -6 \\ 0 & -2 & -1 \\ -1 & 3 & 2 \end{bmatrix}^T = \begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$$

Now, $A^{-1} = \dfrac{\text{adj } A}{|A|} = \dfrac{1}{-1}\begin{bmatrix} 2 & 0 & -1 \\ -9 & -2 & 3 \\ -6 & -1 & 2 \end{bmatrix}$

$$= \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}$$

The matrix form of the given system of equations can be represented as

$$\begin{bmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{bmatrix}\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

$$\Rightarrow \quad A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}$$

Premultiplying both sides by A^{-1}

$$A^{-1}A\begin{bmatrix} x \\ y \\ z \end{bmatrix} = A^{-1}\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \qquad [\because A^{-1}A = I = AA^{-1}]$$

$$\Rightarrow \quad I\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{bmatrix}\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix} \qquad [\because IA = A = AI]$$

$$\Rightarrow \quad \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2+0+3 \\ 9+2-9 \\ 6+1-6 \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

On comparing both sides, we get

$x = 1, y = 2,$ and $z = 1$

10. Given system of equations can be written as,

$$2u + 3v + 10w = 4$$
$$4u - 6v + 5w = 1$$
$$6u + 9v - 20w = 2$$

where, $u = \dfrac{1}{x}, v = \dfrac{1}{y}$ and $w = \dfrac{1}{z}$

The above system of equations can be written in matrix form as below:

$$\begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}\begin{bmatrix} u \\ v \\ w \end{bmatrix} = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

or $\qquad AX = B \qquad$ ...(1)

where, $A = \begin{bmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{bmatrix}, X = \begin{bmatrix} u \\ v \\ w \end{bmatrix}$ and $B = \begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$

$$\therefore \quad |A| = \begin{vmatrix} 2 & 3 & 10 \\ 4 & -6 & 5 \\ 6 & 9 & -20 \end{vmatrix}$$

$$= 2(120-45) - 3(-80-30) + 10(36+36)$$
$$= 2 \times 75 + 3 \times 110 + 10 \times 72$$
$$= 150 + 330 + 720 = 1200 \neq 0$$

So, inverse of A exist.

Now, cofactors of matrix A are as follows:

$$A_{11} = (-1)^{1+1}\begin{vmatrix} -6 & 5 \\ 9 & -20 \end{vmatrix} = (-1)^2(120-45) = 1 \times 75 = 75$$

$$A_{12} = (-1)^{1+2}\begin{vmatrix} 4 & 5 \\ 6 & -20 \end{vmatrix} = (-1)^3(-80-30) = -1 \times -110$$
$$= 110$$

$$A_{13} = (-1)^{1+3}\begin{vmatrix} 4 & -6 \\ 6 & 9 \end{vmatrix} = (-1)^4(36+36) = 1 \times 72 = 72$$

$$A_{21} = (-1)^{2+1}\begin{vmatrix} 3 & 10 \\ 9 & -20 \end{vmatrix} = (-1)^3(-60-90) = -1 \times -150$$
$$= 150$$

$$A_{22} = (-1)^{2+2}\begin{vmatrix} 2 & 10 \\ 6 & -20 \end{vmatrix} = (-1)^4(-40-60) = 1 \times -100$$
$$= -100$$

$$A_{23} = (-1)^{2+3}\begin{vmatrix} 2 & 3 \\ 6 & 9 \end{vmatrix} = (-1)^5(18-18) = -1 \times 0 = 0$$

$$A_{31} = (-1)^{3+1}\begin{vmatrix} 3 & 10 \\ -6 & 5 \end{vmatrix} = (-1)^4(15+60) = 1 \times 75 = 75$$

$$A_{32} = (-1)^{3+2}\begin{vmatrix} 2 & 10 \\ 4 & 5 \end{vmatrix} = (-1)^5(10-40) = -1 \times (-30) = 30$$

$$A_{33} = (-1)^{3+3}\begin{vmatrix} 2 & 3 \\ 4 & -6 \end{vmatrix} = (-1)^6(-12-12) = 1 \times (-24)$$
$$= -24$$

$$\therefore \quad \text{adj } A = \begin{bmatrix} A_{11} & A_{12} & A_{13} \\ A_{21} & A_{22} & A_{23} \\ A_{31} & A_{32} & A_{33} \end{bmatrix}^T = \begin{bmatrix} 75 & 110 & 72 \\ 150 & -100 & 0 \\ 75 & 30 & -24 \end{bmatrix}^T$$

$$= \begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}$$

So, $\quad A^{-1} = \dfrac{\text{adj } A}{|A|}$

$$= \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix} \qquad ...(2)$$

$\because$ From eq. (1),

$AX = B$

or $\quad X = A^{-1}B$

$$\Rightarrow \quad \begin{bmatrix} u \\ v \\ w \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 75 & 150 & 75 \\ 110 & -100 & 30 \\ 72 & 0 & -24 \end{bmatrix}\begin{bmatrix} 4 \\ 1 \\ 2 \end{bmatrix}$$

$$= \frac{1}{1200}\begin{bmatrix} 300+150+150 \\ 440-100+60 \\ 288+0-48 \end{bmatrix} = \frac{1}{1200}\begin{bmatrix} 600 \\ 400 \\ 240 \end{bmatrix}$$

$$\Rightarrow \quad \begin{bmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ \frac{1}{3} \\ \frac{1}{5} \end{bmatrix}$$

Comparing corresponding elements, we get

$$\frac{1}{x} = \frac{1}{2}, \frac{1}{y} = \frac{1}{3}, \frac{1}{z} = \frac{1}{5}$$

$$\Rightarrow \quad x = 2, y = 3, z = 5$$

Chapter Test

Multiple Choice Questions

Q 1. Which of the following is correct?

a. Determinant is a square matrix

b. Determinant is a number associated to a matrix

c. Determinant is a number associated to a square matrix

d. All of the above

Q 2. If A is an invertible matrix of order n, then |adj A| is equal to:

a. $|A|^n$ b. $|A|^{n+1}$

c. $|A|^{n-1}$ d. $|A|^{n+2}$

Assertion and Reason Type Questions

Directions (Q. Nos. 3-4): *In the following questions, each question contains Assertion (A) and Reason (R). Each question has 4 choices (a), (b), (c) and (d) out of which only one is correct. The choices are:*

a. Both Assertion (A) and Reason (R) are true and Reason (R) is the correct explanation of Assertion (A)

b. Both Assertion (A) and Reason (R) are true but Reason (R) is not the correct explanation of Assertion (A)

c. Assertion (A) is true but Reason (R) is false

d. Assertion (A) is false but Reason (R) is true

Q 3. Assertion (A): If $\begin{vmatrix} 2x & 5 \\ 8 & x \end{vmatrix} = \begin{vmatrix} 6 & -2 \\ 7 & 3 \end{vmatrix}$, then $x = \pm 6$.

Reason (R): If A and B are matrices of order 3 and $|A| = 4, |B| = 6$, then $|2AB| = 192$.

Q 4. Assertion (A): Minor of an element of a determinant of order n ($n \geq 2$) is a determinant of order n.

Reason (R): If A is an invertible matrix of order 2, then det (A^{-1}) is equal to $\frac{1}{|A|}$.

Case Study Based Questions

Q 5. Case Study 1

A factory produces three items every day. Their production on certain day is 45 tons. It is found that the production of third items exceeds the production of first item by 8 tons while the total production of first and third item is twice the production of second item.

Based on the given information, solve the following questions:

(i) If x, y and z respectively denote the quantity (in tons) of first, second and third item produced, then find the given conditions algebraically.

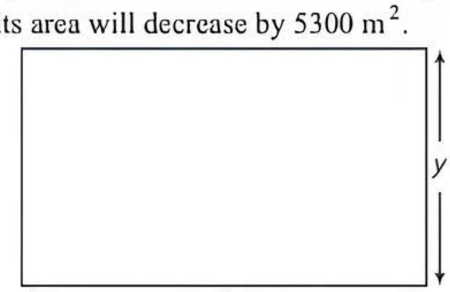
(ii) If $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -2 \\ 1 & -1 & 1 \end{bmatrix}^{-1} = \frac{1}{6}\begin{bmatrix} 2 & 2 & 2 \\ 3 & 0 & -3 \\ 1 & -2 & 1 \end{bmatrix}$, then find the inverse of $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & -1 \\ 1 & -2 & 1 \end{bmatrix}$.

(iii) Find the ratio $x : y : z$.

Q 6. Case Study 2

Manjit wants to donate a rectangular plot of land for a school in his village.

When he was asked to give dimensions of the plot, he told that if its length is decreased by 50 m and breadth is increased by 50 m, then its area will remain same, but if length is decreased by 10 m and breadth is decreased by 20 m, then its area will decrease by 5300 m^2.



Based on the above information, solve the following questions:

(i) Find the algebraic equations in terms of x and y.

(ii) Find the values of x (length of rectangular field) and y (breadth of rectangular field).

(iii) How much is the area of rectangular field?

Or

How much is the perimeter of rectangular field?

Very Short Answer Type Questions

Q 7. If $P = \begin{bmatrix} 10 & -2 \\ -5 & 1 \end{bmatrix}$, then find P^{-1} if exists.

Q 8. If $\begin{bmatrix} 2+x & 3 & 4 \\ 1 & -1 & 2 \\ x & 1 & -5 \end{bmatrix}$ is a singular matrix, then find the value of x.

Short Answer Type Questions

Q 9. Find the cofactors of elements a_{12}, a_{22}, a_{32} respectively of the matrix $\begin{bmatrix} 1 & \sin\theta & 1 \\ -\sin\theta & 1 & \sin\theta \\ -1 & -\sin\theta & 1 \end{bmatrix}$.

Q 10. If the points $(3, -2)$, $(x, 2)$, $(8, 8)$ are collinear, then find the value of x.

Q 11. For $A = \begin{bmatrix} 3 & -4 \\ 1 & -1 \end{bmatrix}$, write A^{-1}.

Q 12. If $A = \begin{bmatrix} 0 & 1 & 3 \\ 1 & 2 & x \\ 2 & 3 & 1 \end{bmatrix}$ and $A^{-1} = \begin{bmatrix} 1/2 & -4 & 5/2 \\ -1/2 & 3 & -3/2 \\ 1/2 & y & 1/2 \end{bmatrix}$, then find the value of x and y.

Long Answer Type Questions

Q 13. If $A = \begin{bmatrix} 1 & 2 & 2 \\ 2 & 1 & 2 \\ 2 & 2 & 1 \end{bmatrix}$, then find A^{-1}. Also verify that $AA^{-1} = A^{-1}A = I$.

Q 14. If $A = \begin{bmatrix} 2 & -3 & 5 \\ 3 & 2 & -4 \\ 1 & 1 & -2 \end{bmatrix}$, then find the inverse of matrix A. Also find the solution of the following system equations:

$$2x - 3y + 5z = 11,$$
$$3x + 2y - 4z = -5,$$
$$x + y - 2z = -3$$